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Some Fixed Point Results for $\mathcal{F}-G-$Contraction in $\mathcal{F}-$Metric Spaces Endowed with a Graph

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Abstract. In this paper, we introduce the concept of $\mathcal{F}-G-$contraction mappings in $\mathcal{F}$-metric spaces endowed with a graph and give some fixed point results for such contractions. Our results are generalization of some famous theorem in metric spaces to $\mathcal{F}-$metric spaces endowed with a graph. Also, we give some examples that support obtained theoretical results.

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1 Introduction

Fixed point theory is one of the traditional theory in functional and nonlinear analysis. Fixed point theory has developed rapidly in various extensions of metric spaces (see e.g. [4, 6, 9, 11, 14, 15, 20, 21, 22, 25]

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and references therein). Jleli and Samet [24] introduced the concept of a $\mathcal{F}$-metric spaces as follows (see e.g. [18, 26] and references therein).

Let $\mathcal{F}$ be the set of functions $f : (0,\infty) \to \mathbb{R}$ such that
($\mathcal{F}_1$) f is non-decreasing, i.e., $0 < s < t$ implies $f(s) \leq f(t)$.
($\mathcal{F}_2$) For every sequence $\{t_n\} \subset (0,\infty)$, we have

$$\lim_{n\to\infty} t_n = 0 \text{ if and only if } \lim_{n\to\infty} f(t_n) = -\infty.$$

Definition 1.1. [24] Let X be a (nonempty) set. A function $D : X \times X \to [0,\infty)$ is a $\mathcal{F}$-metric on X iff, there exists $(f,\alpha) \in \mathcal{F} \times [0,\infty)$ such that for all $x, y \in X$ the following conditions are satisfied:
(D_1) $D(x,y) = 0$ if and only if $x = y$.
(D_2) $D(x,y) = D(y,x)$.
(D_3) For every $N \in \mathbb{N}, N \geq 2$ and for every $\{u_i\}_{i=1}^N \subset X$ with $(u_1, u_N) = (x,y)$, we have

$$D(x,y) > 0 \text{ implies } f(D(x,y)) \leq f(\sum_{i=1}^{N-1} D(u_i, u_{i+1})) + \alpha.$$

The pair (X, D) is called a $\mathcal{F}$-metric space.

Example 1.2. [24] Let $X = \mathbb{R}$ and $D : X \times X \to [0,\infty)$ be defined as follows:

$$D(x,y) = \begin{cases} (x-y)^2 & (x,y) \in [0,3] \times [0,3], \\ |x-y| & \text{otherwise,} \end{cases}$$

and let $f(t) = ln(t)$ for all $t > 0$ and $\alpha = ln(3)$. Then, D is a $\mathcal{F}$-metric on X. Since $D(0,3) = 9 \geq D(0,1) + D(1,3) = 5$, then D is not a metric on X.

Example 1.3. [24] Let $X = \mathbb{R}$ and $D : X \times X \to [0,\infty)$ be defined as follows:

$$D(x,y) = \begin{cases} e^{|x-y|} & x \neq y, \\ 0 & x = y. \end{cases}$$

Then, D is a $\mathcal{F}$-metric on X. Since $D(2,4) = e^2 \geq D(2,3) + D(3,4) = 2e$, so D is not a metric on X.

Definition 1.4. [24] Let (X, D) be an $\mathcal{F}$-metric space and $\{x_n\}$ be a sequence in X.
1) A sequence $\{x_n\}$ is called $\mathcal{F}$-convergent to $x \in X$, iff $D(x_n, x) \to 0$ as $n \to \infty$.
2) A sequence $\{x_n\}$ is $\mathcal{F}$-Cauchy, iff $D(x_n, x_m) \to 0$ as $n, m \to \infty$.
3) A $\mathcal{F}$-metric space (X, D) is said to be $\mathcal{F}$-complete, if every $\mathcal{F}$-Cauchy sequence in X is $\mathcal{F}$-convergent to some element in X.

Theorem 1.5. *[24] Let (X, D) be an $\mathcal{F}$-complete $\mathcal{F}$-metric space and let $T : X \to X$ be a self-mapping satisfying*

$$D(Tx, Ty) \leq \lambda D(x, y), \tag{1}$$

for all $x, y \in X$ where $0 \leq \lambda < 1$. Then T has a unique fixed point.

Espinola and Kirk in 2006 published some useful results on combining fixed point theory and graph theory [12]. In 2008, Jachymski [23] proved the contraction Principal for mappings on a metric space with a graph. For some recent works in metric spaces endowed with graph the reader is referred to (see e.g. [1, 2, 3, 5, 7, 8, 10, 13, 16, 17, 19, 28]

Let $G = (V(G), E(G))$ be a directed graph such that $V(G)$ is the set of vertices and $E(G)$ is edges of G. Also $\Delta \subset E(G)$ where $\Delta = \{(x, x) : x \in X\}$ and assume that G has no parallel edges. We denote the conversion of a graph G by G^{-1}, i.e., the graph obtained from G by reversing the direction of edges. Let $\tilde{G}$ be the undirected graph obtained from G by ignoring the direction of edges, so we have $E(\tilde{G}) = E(G) \bigcup E(G^{-1})$. Let x and y are vertices in a graph G. A path in G from x to y of length m is a sequence $\{x_n\}_{n=0}^{m}$ of $m+1$ vertices such that $x_0 = x, x_m = y$ and $(x_{i-1}, x_i) \in E(G)$ for $i = 1, ..., m$. A graph G is called connected if there is a path between any two vertices of G and graph G is weakly connected if $\tilde{G}$ is connected. For $x \in X$ we set $[x]_{\tilde{G}}$ which is the equivalence class of the following relation R defined on $V(G)$ by the rule: xRy if there is a path in G from x to y. Also, for $x \in G$ and $m \in \mathbb{N}$, define

$$[x]_G^m = \{y \in X : \text{there is a directed path from } x \text{ to } y \text{ of length } m\}.$$

Definition 1.6. [27] Let (X, d) be a metric space and $T : X \to X$ be a self-mapping. Then

i) T is called a Picard operator (briefly PO), if T has a unique fixed point $x^* \in X$ and $T^n x \to x^*$ for each $x \in X$.

ii) T is called a weakly Picard operator (briefly WPO) if the sequence $\{T^n x\}$ converges to a fixed point of T for all $x \in X$.

Definition 1.7. [23] Let (X, d) be a metric space endowed with a graph G. A mapping $T : X \to X$ is called orbitally $G-$continuous on X if for all $x, y \in X$ and all $\{p_n\}$ of positive integers with $(T^{p_n} x, T^{p_n+1} x) \in E(G)$ for all $n \geq 1$, the convergence $T^{p_n} x \to y$ implies $T(T^{p_n} x) \to Ty$.

Let T be a self mapping on X. We denote

$$X_T = \{x \in X | (x, Tx) \in E(G)\},$$

$$Fix(T) = \{x \in X | Tx = x\}.$$

2 Main Results

Now, we introduce one new type of contractive mappings in the context of $\mathcal{F}$-metric spaces endowed with a graph and prove the corresponding new result. We also prove and extend some the results of Jachymski [23] and Falahi et al. [13] to the context of $\mathcal{F}-$metric spaces. Throughout this section we assume that (X, D) is a $\mathcal{F}-$metric space endowed with directed graph G, which $V(G) = X$ and $\Delta \subset E(G)$.

Definition 2.1. Let (X, D) be an $\mathcal{F}$-metric space and T be a self-mapping on X. We say that T is an $\mathcal{F} - G-$contraction if for every $x, y \in X$, we have

$$(x, y) \in E(G) \text{ implies } \quad (Tx, Ty) \in E(G);$$

$$(x, y) \in E(G) \text{ implies } \quad D(Tx, Ty) \leq \lambda D(x, y);$$

where $\lambda \in [0, 1)$.

Example 2.2. Let $(X, \mathcal{F})$ be an $\mathcal{F}-$metric space and $G = (X, \Delta)$. Then any self-mapping T on X is an $\mathcal{F} - G-$contraction.

Example 2.3. Let X be a nonempty set and $(X,\mathcal{F})$ be an $\mathcal{F}-$metric space. Then for any graph $G=(X,E(G))$, constant mapping $T:X\to X$ is a $\mathcal{F}-G-$contraction.

Example 2.4. Consider the $\mathcal{F}$-metric space given in Example 1.2. Define

$$Tx=\begin{cases}3x & x>2\\ \frac{x}{2} & 0\leq x\leq 2\\ 0 & x<0.\end{cases}$$

Then, for any $\lambda\in[0,1)$, we have

$$D(T2,T3)=D(\frac{2}{3},9)=|\frac{2}{3}-9|=\frac{25}{3}>\lambda=\lambda D(2,3).$$

Then, T does not satisfy (1). Define $G=(V(G),E(G))$, where $V(G)=\mathbb{R}$ and $E(G)=\{(x,x)|x\in\mathbb{R}\}$. Therefore, T is an $\mathcal{F}-G-$contraction mapping for any $\lambda\in[0,1)$.

Example 2.5. Let $X=\{0,1,2\}$ be endowed with the $\mathcal{F}$-metric given in Example 1.3. Define $T0=T2=0, T1=2$. Then, for any $\lambda\in[0,1)$, we have

$$D(T1.T2)=e^{|T1-T2|}=e^2>\lambda e=\lambda D(1,2).$$

Consequently, T does not satisfy (1). Define $G=(V(G),E(G))$, where $V(G)=X$ and $E(G)=\{(0,0),(1,1),(0,2),(2,2)\}$. Then T is an $\mathcal{F}-G-$contraction mapping for any $\lambda\in[0,1)$.

Proposition 2.6. *Let (X,D) be an $\mathcal{F}-$metric space and $T:X\to X$ be a $\mathcal{F}-G-$contraction. Then:*
(i) T is a $\mathcal{F}-\tilde{G}-$contraction and also a $\mathcal{F}-G^{-1}-$contraction.
(ii) $[x_0]_{\tilde{G}}$ is $T-$invariant and $T|_{[x_0]_{\tilde{G}}}$ is a $\mathcal{F}-\tilde{G}_{x_0}-$contraction, where $x_0\in X$ and $T(x_0)\in[x_0]_{\tilde{G}}$.

Proof. (i) Since $\mathcal{F}-$metric is symmetric, then T is a $\mathcal{F}-\tilde{G}-$contraction and also a $\mathcal{F}-G^{-1}-$contraction.
(ii) Let $x\in[x_0]_{\tilde{G}}$. So there exists a path $\{z_i\}_{i=0}^N$ in $\tilde{G}$ from x to x_0 which $x=z_0$ and $x_0=z_N$ and $(z_{i-1},z_i)\in E(\tilde{G})$. Since T is a

$\mathcal{F}-G-$contaraction, for all $i=1,...,N$, we have $(Tz_{i-1},Tz_i)\in E(G)$. Then $Tx\in[Tx_0]_{\tilde{G}}=[x_0]_{\tilde{G}}$, that is, $[x_0]_{\tilde{G}}$ is $T-$invariant. Now, assume $(x,y)\in E(\tilde{G}_{x_0})$. Since T is a $\mathcal{F}-G-$contraction, $(Tx,Ty)\in E(G)$. Also, $[x_0]_{\tilde{G}}$ is T-invariant, then $(Tx,Ty)\in E(\tilde{G}_{x_0})$. Since $\tilde{G}_{x_0}$ is a subgraph of G, we obtain $T|_{[x_0]_{\tilde{G}}}$ is a $\mathcal{F}-\tilde{G}_{x_0}-$contraction. □

Definition 2.7. Let $(X,\mathcal{F})$ be a $\mathcal{F}-$metric space. We say that sequences $\{x_n\},\{y_n\}$ are equivalent if $\lim_{n\to\infty}D(x_n,y_n)=0$, and they are called $\mathcal{F}-$Cauchy equivalent, if each of them is a $\mathcal{F}-$Cauchy sequence.

The following result extend the main one from [23].

Theorem 2.8. *Let (X,D) be an $\mathcal{F}$-metric space. The following are equivalent:*

(i) *G is weakly connected.*

(ii) *For any $\mathcal{F}-G-$contraction $T:X\to X$, given $x,y\in X$, the sequences $\{T^nx\}$ and $\{T^ny\}$ are equivalent.*

iii) *For any $\mathcal{F}-G-$contraction $T:X\to X$, $card(Fix(T))\leq 1$.*

Proof. First we prove that (i) implies (ii). Let $x,y\in X$ and by hypothesis, $[x]_{\tilde{G}}=X$, then $y\in[x]_{\tilde{G}}$. So there exists a path $\{x_i\}_{i=0}^N$ in $\tilde{G}$ from x to y which $x_0=x$ and $x_N=y$ and $(x_{i-1},x_i)\in E(\tilde{G})$ for all $i=1,2,...,N$. Using Proposition 2.6, T is an $\mathcal{F}-\tilde{G}-$contraction. Then, we have

$$(T^nx_{i-1},T^nx_i)\in E(\tilde{G}),$$

consequently

$$D(T^nx_{i-1},T^nx_i)\leq\lambda D(T^{n-1}x_{i-1},T^{n-1}x_i),$$

for all $n\in\mathbb{N}$ and $i=1,...,N$. Then, we get

$$D(T^nx_{i-1},T^nx_i)\leq\lambda^nD(x_{i-1},x_i),\tag{2}$$

for all $n\in\mathbb{N}$ and $i=1,...,N$. Now, let $(f,\alpha)\in\mathcal{F}\times[0,+\infty)$ be such that (D_3) is satisfied and $\varepsilon>0$ be fixed. From $(\mathcal{F}_2)$, there exists $\delta>0$ such that

$$0<t<\delta \text{ implies } f(t)<f(\varepsilon)-\alpha.\tag{3}$$

Using (2), we have

$$\sum_{i=1}^{N} D(T^n x_{i-1}, T^n x_i) \leq \sum_{i=1}^{N} \lambda^n D(x_{i-1}, x_i) = \lambda^n \sum_{i=1}^{N} D(x_{i-1}, x_i).$$

Scince $\lim_{n\to\infty} \lambda^n \sum_{i=1}^{N} D(x_{i-1}, x_i) = 0$, there exists some $N_0 \in \mathbb{N}$ such that

$$0 < \lambda^n \sum_{i=1}^{N} D(x_{i-1}, x_i) < \delta, \quad n \geq N_0.$$

Using (3) and $(\mathcal{F}_1)$, we obtain

$$f(\sum_{i=1}^{N} D(T^n x_{i-1}, T^n x_i)) \leq f(\lambda^n \sum_{i=1}^{N} D(x_{i-1}, x_i)) < f(\varepsilon) - \alpha, \tag{4}$$

for all $n \geq N_0$. Using (D_3) and (4), we have

$$f(D(T^n x, T^n y)) \leq f(\sum_{i=1}^{N} D(T^n x_{i-1}, T^n x_i)) + \alpha \leq f(\varepsilon) - \alpha + \alpha < f(\varepsilon),$$

for all $n \geq N_0$. Then, we get

$$D(T^n x, T^n y) < \varepsilon, \quad n \geq N_0.$$

So $D(T^n x, T^n y) \to 0$ as $n \to \infty$, that is, the sequences $\{T^n x\}$ and $\{T^n y\}$ are equivalent.
Now, we shall prove that (ii) implies (iii). Let T be a $\mathcal{F}-G-$contraction and $x, y \in Fix(T)$. From (ii), $\{T^n x\}$ and $\{T^n y\}$ are equivalent. Then, we have $D(x, y) = D(T^n x, T^n y) \to 0$ as $n \to \infty$, that is, $x = y$.
Finally we prove that (iii) implies (i). On the contrary, we assume that G is not weakly connected, that is, $\tilde{G}$ is disconnected. Suppose that there exists $x_0 \in X$ such that both sets $[x_0]_{\tilde{G}}$ and $X - [x_0]_{\tilde{G}}$ are nonempty. Suppose $y_0 \in X - [x_0]_{\tilde{G}}$ and define

$$Tx = x_0 \text{ if } x \in [x_0]_{\tilde{G}} \quad ; \quad Tx = y_0 \text{ if } x \in X - [x_0]_{\tilde{G}}.$$

Consequently, $Fix(T) = \{x_0, y_0\}$. Now, we show that T is a $\mathcal{F}-G-$contraction. Suppose $(x, y) \in E(G)$, so $[x]_{\tilde{G}} = [y]_{\tilde{G}}$, that is, $x, y \in$

$[x_0]_{\tilde{G}}$, or $x, y \in X - [x_0]_{\tilde{G}}$. Then, we have $Tx = Ty$, so $(Tx, Ty) \in E(G)$. Since $\Delta \subset E(G)$ and $D(Tx, Ty) = 0 \leq \lambda D(x, y)$ for any $\lambda \in [0, 1)$, we get T is a $\mathcal{F} - G-$contraction having two fixed points which violates assumption (iii). □

Corollary 2.9. *Let (X, D) be an $\mathcal{F}$-complete $\mathcal{F}$-metric space endowed with a graph weakly connected G. Then, for any $\mathcal{F} - G-$contraction $T : X \to X$, there is $x^* \in X$ such that $\lim_{n\to\infty} T^n x = x^*$ for all $x \in X$.*

Proof. Let $T : X \to X$ be a $\mathcal{F} - G-$contraction and fix any point $x \in X$. Let $m > n \geq 0$ and $m, n \in \mathbb{N}$. Scince G is a weakly connected, from Theorem 2.8, the sequences $\{T^n x\}$ and $\{T^n T^{m-n} x\}$ are equvailent. Then $\lim_{n,m\to\infty} D(T^n x, T^m x) = 0$, that is, $\{T^n(x)\}$ is a $\mathcal{F}$-Cauchy sequence in X. Hence, there exists $x^* \in X$ such that $T^n x \to x^*$ as $n \to \infty$. Suppose $y \in X$, then by Theorem 2.8, sequencs $\{T^n x\}$ and $\{T^n y\}$ are equivalent. Using (D_3), we have

$$f(D(T^n y, x^*) \leq f(D(T^n x, T^n y) + D(T^n x, x^*)) + \alpha,$$

for all $n \in \mathbb{N}$. Since $D(T^n x, T^n y) + D(T^n x, x^*) \to 0$ as $n \to \infty$, so $\lim_{n\to\infty} f(D(T^n x, T^n y) + D(T^n x, x^*)) + \alpha = -\infty$. Then $D(T^n y, x^*) \to 0$ as $n \to \infty$. □

Theorem 2.10. *Let (X, D) be an $\mathcal{F}-$complete $\mathcal{F}-$metric space endowed with a graph G and T be a self-mapping on X such that T is a $\mathcal{F} - G-$contraction mapping. Then $T|_{X_T}$ is a weakly Picard operator if one of the following conditions hold:*

i) *T is orbitally G-continuous on X.*

ii) *If $x_n \to x$ as $n \to \infty$ and $(x_n, x_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $(x_{n_k}, x) \in E(G)$ for all $k \in \mathbb{N}$.*

Moreover, if (i) or (ii) holds, then $X_T \neq \emptyset$ if and only if $Fix(T) \neq \emptyset$.

Proof. If $X_T = \emptyset$, then it is clear that there is nothing to prove. Let $x \in X_T$, then $(x, Tx) \in E(G)$ and since T is an $\mathcal{F} - G-$contraction mapping, it following $(Tx, T^2 x) \in E(G)$, that is, $Tx \in X_T$. Thus, T

maps X_T into X_T. Then, it follows by induction that $(T^n x, T^{n+1}x) \in E(G)$ and

$$D(T^n x, T^{n+1}x) \leq \alpha^n D(x, Tx), \tag{5}$$

for all $n \in \mathbb{N}$. Let $(f, \alpha) \in \mathcal{F} \times [0, +\infty)$ be such that (D_3) is satisfied and $\varepsilon > 0$ be fixed. Using $(\mathcal{F}_2)$, there exists $\delta > 0$ such that

$$0 < t < \delta \text{ implies } f(t) < f(\varepsilon) - \alpha. \tag{6}$$

From (5), we have

$$\sum_{i=n}^{m} D(T^i x, T^{i+1}x) \leq \sum_{i=n}^{m} \lambda^i D(x, Tx) \leq \frac{\lambda^n}{1-\lambda} D(x, Tx),$$

for all $m \geq n \geq 0$. Scince $\lim_{n\to\infty} \frac{\lambda^n}{1-\lambda} D(x, Tx) = 0$, there exists some $N_0 \in \mathbb{N}$ such that

$$0 < \frac{\lambda^n}{1-\lambda} D(x, Tx) < \delta, \quad n \geq N_0.$$

Using (6) and $(\mathcal{F}_1)$, we have

$$f(\sum_{i=n}^{m} D(T^i x, T^{i+1}x)) \leq f(\frac{\lambda^n}{1-\lambda} D(x, Tx)) < f(\varepsilon) - \alpha. \tag{7}$$

Then, from (D_3) and (7), we get

$$f(D(T^m x, T^n x)) \leq f(\sum_{i=n}^{m} D(T^i x, T^{i+1}x)) + \alpha < f(\varepsilon).$$

Using $(\mathcal{F}_1)$, we obtain

$$D(T^m x, T^n x) < \varepsilon, \qquad m > n \geq N_0.$$

This prove that $\{T^n x\}$ is a $\mathcal{F}-$Cauchy sequence. Since (X, D) is $\mathcal{F}$-complete, there exists $x^* \in X$, such that

$$\lim_{n\to\infty} T^n x = x^*. \tag{8}$$

Now, we show that x^* is a fixed point of T. To this end, if T is orbitally G-continuous on X, then $T^{n+1}x \to Tx^*$ as $n \to \infty$. Because the limit of convergent sequence in a $\mathcal{F}$-metric space is unique, we get, $Tx^* = x^*$. Now, we suppose that condition (ii) holds. Then there exists a strictly increasing sequence $\{n_k\}$ of positive integer such that $(T^{n_k}x, x^*) \in E(G)$ for all $k \geq 1$. Then, from (D_3), we have

$$\begin{aligned} f(D(Tx^*, x^*)) &\leq f(D(Tx^*, T^{n_k+1}x) + D(T^{n_k+1}x, x^*)) + \alpha \\ &\leq f(\lambda D(x^*, T^{n_k}x) + D(T^{n_k+1}x, x^*)) + \alpha \end{aligned}$$

Using $(\mathcal{F}_2)$ and (8), we have

$$\lim_{k\to\infty} f(\lambda D(x^*, T^{n_k}x) + D(T^{n_k+1}x, x^*)) + \alpha = -\infty,$$

which is a contradiction. Therefore, we have $D(Tx^*, x^*) = 0$, i.e. $Tx^* = x^*$. Since $Fix(T) \subset X_T$, we have $x^* \in X_T$, that is, $T|_{X_T}$ is a weakly Picard operator. □

In Theorem 2.10, if $G = G_0$, where $G_0 = (X, X \times X)$, then $X_T = X$ and we get the following corollary.

Corollary 2.11. *Let (X, D) be a $\mathcal{F}$-complete $\mathcal{F}-$metric space and T be a self-mapping on X which satisfy (1). Then T is a Picard operator.*

References

[1] M. Abbas, T. Nazir and H. Aydi, Fixed points of generalized graphic contraction mappings in partial metric spaces endowed with a graph, *J. Adv. Math. Stud.,* 6(2), (2013), 130-139.

[2] M. Abbas, T. Nazir, B. Popovíć and S. Radenović, On weakly commuting set-valued mappings on a domain of sets endowed with directed graph, *Results Math.,* 71(3-4), (2017), 1277-1295.

[3] M. Abbas, T. Nazir, T. Aleksić Lampert and S. Radenović, Common fixed points of set-valued F-contraction mappings on domain of sets endowed with directed graph, *Comput. Appl. Math.,* 36(4), (2017), 1607-1622.

[4] S. P. Acharya, Some results on fixed points in uniform spaces, *Yokohama Math. J.,* 22, (1974), 105-116.

[5] S. M. A. Aleomraninejad, Sh. Rezapour and N. Shahzad, Some fixed point results on a metric space with a graph, *Topology Appl.,* 159(3) (2012), 659-663.

[6] M. A. Alghamdi, S. Gulyaz-Ozyurt and E. Karapınar, A note on extended Z-contraction, *Mathematics*, 8(2) (2020), p. 195.

[7] M. U. Ali, Fahimuddin, T. Kamran and E. Karapınar, Fixed point theorems in uniform space endowed with graph, *Miskolc Mathematical Notes*, 18(1) (2017), pp. 57–69.

[8] A. Azam, N. Mehmood, T. Došenović and S. Radenović, Coincidence point of L-fuzzy sets endowed with graph, *RACSAM*, (2018), 112:915-931.

[9] I. C. Chifu and E. Karapınar, Admissible Hybrid Z-Contractions in b-Metric Spaces, *Axioms*, 9(1), (2020), 2.

[10] C. Chifu, E. Karapınar and G. Petrusel, Qualitative properties of the solution of a system of operator inclusions in b-metric spaces endowed with a graph, *Bull. Iranian Math. Soc.*, 44(5) (2018), 1267-1281.

[11] L. B. Ćirić, *Some Recent Results In Metrical Fixed Point Theory*, Faculty of Mechanical Engineering, University of Belgrade, Belgrade (2003).

[12] R. Espinola and W.A. Kirk, Fixed point theorems in R-trees with applications to graph theory, *Topology Appl.,* 153, (2006), 1046-1055.

[13] K. Fallahi and A. Aghanians, Fixed points for Chatterjea contractions on a metric space with a graph, *Int. J. Nonlinear Anal. Appl.,* 7(2), (2016), 49-58.

[14] H. Faraji, K. Nourouzi and D. O'Regan, A fixed point theorem in uniform spaces generated by a family of b-pseudometrics, *Fixed Point Theory*, 20(1), (2019), 177-183.

[15] H. Faraji, D. Savić and S. Radenović, Fixed point theorems for Geraghty contraction type mappings in b-metric spaces and applications, *Axioms*, 8(1), (2019), 34.

[16] R. George, H. A. Nabwey, R. Ramaswamy and S. Radenović, Some generalized contraction classes and common fixed points in b-metric space endowed with a graph, *Mathematics*, 7(8), (2019), 754; doi:10.3390/math7080754.

[17] G. Gwozdz-Lukawska and J. Jachymski, IFS on a metric space with a graph structure and extensions of the Kelisky-Rivlin theorem, *J. Math. Anal. Appl.*, 356(2), (2009), 453-463.

[18] A. Hussain and T. Kanwal, Existence and uniqueness for a neutral differential problem with unbounded delay via fixed point results, *Trans. A. Razmadze Math. Inst.*, 172(3), (2018), 481-490.

[19] T. Kamran, M. Samreen and N. Shahzad, Probabilistic G-contractions, *Fixed Point Theory Appl.*, 2013(1), (2013), 223, 14.

[20] E. Karapinar and C. Chifu, Results in wt-Distance over b-Metric Spaces, *Mathematics*, 8(2), (2020), 220.

[21] E. Karapinar, A. Fulga and A. Petrusel, On Istratescu Type Contractions in b-Metric Spaces, *Mathematics*, 8(3), (2020), 388.

[22] W. Kirk and N. Shahzad, *Fixed Point Theory In Distances Spaces*, Springer: Berlin, Germany, (2014).

[23] J. Jachymski, The contraction principal for mappings on a metric space with a graph, *Proc. Amer. Math. Soc.*, 36(4), (2008), 1359-1373.

[24] M. Jleli and B. Samet, On a new generalization of metric spaces, *J. Fixed Point Theory Appl.*, 20(3), (2018), Art. 128, 20 pp.

[25] S.G. Matthews, *Partial Metric Topology*, Research Report 212, Dept. of Computer Science, University of Warwick, (1992).

[26] Z. D. Mitrović, H. Aydi, N. Hussain and A. Mukheimer, Reich, Jungck, and Berinde common fixed point results on $\mathcal{F}$-metric spaces and an application, *Mathematics*, (2019), 7, 387.

[27] A. Petrusel and I.A. Rus, Fixed point theorems in ordered L-spaces, *Proc. Amer. Math. Soc.*, 134(2), (2006), 411-418.

[28] M. D. L. Sen, N. Nikolić, T. Došenović, M. Pavlović and S. Radenović, Some results on $(sq)-$graphic contraction mappings in b-metric-like spaces, *Mathematics*, 7(12), (2019), 1190; doi:10.3390/math7121190.

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