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On Time Reversibility of Linear Time Series

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Abstract. In this paper we suggest a procedure for testing reversibility of time series. Our approach is based on a necessary and sufficient condition for time reversibility of linear models. An attractive feature of the procedure is that in converse with other approaches it doesn't require important assumptions, especially existence of moments of order higher than two. Our simulation confirms the procedure and some empirical examples are given.

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1. Introduction

Time reversibility of a stochastic process means that each part of the process has the same probabilistic properties as its time reversal. In other words, a stochastic process $\{X_t,\ t \in \mathbb{R}\}$ is called time reversible if for each $t_0, t_1, \ldots, t_n$ and τ in $\mathbb{R}$, $(X_{t_0}, X_{t_1,\ldots,}X_{t_n})$ and $(X_{\tau-t_0}, X_{\tau-t_1}, ..., X_{\tau-t_n})$ have the same joint probability distribution. We denote this by

$$(X_{t_0}, X_{t_1,\ldots,}X_{t_n}) \stackrel{d}{=} (X_{\tau-t_0}, X_{\tau-t_1}, ..., X_{\tau-t_n})\,.$$

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There are many examples of commonly used statistical models for discrete time stochastic processes that are time reversible, including sequence of independent identical distributed random variables or even stationary process of independent random variables and stationary Gaussian processes. On the other hand, a linear, non Gaussian process is time irreversible in general, except when its coefficients satisfy a very restrictive condition. Therefore with a knowledge about time reversibility, for finding best fitted model, we can seek in a much smaller class and it is not surprising that testing for reversibility is important for model building in the time series analysis. Due to time reversibility is a necessary condition for an independent and identically distributed sequence, several tests for time reversibility have been suggested to be applied as tests for model misspecification. A test for time reversibility could be used before the assumption of a linear Gaussian model can be made, or before a point transformation to a linear Gaussian model can be attempted. Moreover, if a stationary linear model autoregressive moving average (ARMA) is assumed, then mostly a test for time reversibility is equivalent to a test for Gaussianity.

Ramsey and Rothman (1996) survey much of this literature. They provided a connection between aspects of asymmetry and concepts of time reversibility and irreversibility. They used it to introduce a statistical tool for identifying time irreversible stochastic processes that was named the symmetric-bicovariance function. In Ramsey-Rothman test (RRT), one has to calculate a complicated and asymptotic estimator for variance of test statistic by fitting the data to an ARMA model, which is a dark point, then obtaining an estimate of the innovations variance and simulating a time series using the estimated ARMA coefficient values and generating a Gaussian distribution with zero mean and variance equal to that estimated value. This test examines the behavior of estimated third order moments to check for departures from time reversibility and it is applicable only if the one dimensional marginal law of the process has a finite sixth moment. Such a condition may be too restrictive and rules out time series.

Also Ramsey and Rothman have used this point that a time series can be reversible only if the model is nonlinear time reversible or autoregressive moving average (ARMA) with Gaussian innovation, which is not completely true. Although the other situation happens rarely.

Hinich and Rothman(1998) introduced a frequency-domain test of time reversibility based on the bispectrum and called it the Reverse test. This test exploits a property of higher-order spectra for time reversible processes, i.e., the imaginary part of all polyspectra is zero for time reversible stochastic processes. In particular, it checks whether the breakdown of Gaussianity is due to time reversibility or not.

Both Reverse test and RRT examine the behavior of estimated third order moments to check for departures from time reversibility and they are applicable only if the one dimensional marginal law of the data has a finite sixth moment. This requirement eliminates many economic and financial time series, since it is often argued that, although the unconditional variance of such time series exists, their higher order unconditional moments may not be finite.

Cheng (1999) proved a basic theorem which gave a necessary and sufficient condition for time reversibility of stationary linear processes and did not require existence of moments of order higher than two.

Chen et al. (2000) proposed a class of tests for time reversibility, based on a necessary condition for time reversibility that did not have any moment restriction. The proposed test was based on the implication that the differences of the series being tested have symmetric marginal distributions. By contrast, RRT focused only on the third moment of this distributions. They proposed a class of tests for time reversibility based on characteristic functions and suggested different ways to use it.

Franch and Contreras (2004) have compared the power of RRT and Chen et al. (2000) test (CCKT) for their own purpose, but surprisingly in all considered cases, RRT is more powerful than CCKT. Therefore, if we estimate maximum moment exponent of the residuals and this estimate is larger than five, RRT seems the correct choice, otherwise, which in financial data happens usually, CCKT is preferable.

Similar to CCKT, Psaradakis (2008) in his article exploits the fact that time reversibility of stochastic processes implies symmetry of the one dimensional marginal law of differences of the process. His procedure are based on the necessary condition for time reversibility that is an

index of the deviation from zero of the median of the one dimensional marginal law of differences of data. He has considered using subsampling and resampling methods to construct confidence intervals and hypothesis tests for time reversibility.
In this paper, a new procedure for testing time reversibility of a time series is given. This procedure is based on fitting a linear model to data and is according to a necessary and sufficient condition for reversibility of stationary linear processes introduced by Cheng (1999).
The paper proceeds as follow. In Section 2 we introduce some notations and give some preliminary results. In Section 3 the new procedure for testing time reversibility is introduced. Simulation results are presented in Section 4. In Section 5 the time reversibility procedure is used for international real data. At the end, we will provide concluding discussion.

2. Stationary and Linearity

It is not hard to show that each time reversible process is stationary, but the converse is not true. To show time reversibility of a discrete time stationary stochastic process $\{X_n : n \in \mathbb{Z}\}$ it is enough to show that for every positive integer n,

$$(X_0, X_1, ..., X_n) \overset{d}{=} (X_n, X_{n-1}, ..., X_0) .$$

Where $\mathbb{Z}$ is the set of all integers. In this paper here and everywhere else, we assume that the stochastic process involved is discrete time and stationary.
A stochastic process $\{X_n : n \in \mathbb{Z}\}$ is called linear process if it has the following representation

$$X_n = \sum_{i \in \mathbb{Z}} b_i Z_{n-i}, \tag{2.1}$$

where $\{Z_n : n \in \mathbb{Z}\}$ is a sequence of nondegenerate independent and identically distributed random variables with $E(Z_n) = 0$, $0 < E\left(Z_n^2\right) = \sigma^2 < \infty$ and $\{b_i\}$ is a sequence of constants such that $0 < \sum_{i \in \mathbb{Z}} b_i^2 < \infty$.

Note that the concept of time reversibility is distinct from linearity and from the concept of being stationary.

It is obvious that the Gaussian ARMA processes are time reversible. Therefore a statistical test for time reversibility seems to be needed in the analysis of time series. Such a test is required before the assumption of a linear Gaussian model can be made. Moreover, if a stationary linear model ARMA is assumed, then a test for time reversibility is mostley equivalent to a test for the assumption of Gaussianity. Weiss(1975) has shown that time reversibility is mostly unique property of Gaussian processes for ARMA processes and only time reversible non Gaussian ARMA processes are pure moving average (MA) processes, with a restrictive condition on its coefficients, which is usually not the case.

The next theorem extends the result of Weiss and gives a necessary and sufficient condition for time reversibility without requiring the existence of moments higher than second order and is a restatment of Cheng (1999), Theorem 2.

Theorem 2.1. *Let $\{X_n : n \in \mathbb{Z}\}$ be a stationary linear process as $X_n = \sum_{i\in\mathbb{Z}} b_i Z_{n-i}$. $\{X_n : n \in \mathbb{Z}\}$ is time reversible if and only if it is a Gaussian process.or for a constant integer n_0 and $a = +1$ or -1, the following conditions hold.*

i) $b_n = ab_{n_0-n}$.

ii) *Z_n and aZ_n have the same distributions.*

Remark 2.2. *Note that if $b_n = 0$ for $n < 0$, from (i) we have $b_n = ab_{n_0-n}$ for $0 \leqslant n \leqslant n_0$ and vanishes otherwise. Then in this case $\{X_n : n \in \mathbb{Z}\}$ is a special moving average model with finite order not greater than n_0 as $X_n = \sum_{i=0}^{n_0} b_j Z_{n-j}$, we know this model as $MA(n_0)$.*

A consequence of Theorem 2.1 is that the time reversible non Gaussian ARMA processes are small subclasses of moving average non Gaussian processes.

In the next section using Theorem (2.1), we introduce a procedure for testing time reversibility of a given time series.

3. A Procedure for Testing Time Reversibility

In this procedure, we substitute a MA representation as a local approximation to the unknown model for a realization of the stationary stochastic process $\{X_n : n \in \mathbb{Z}\}$ that is $\{x_0, x_1, ..., x_n\}$ and estimate the coefficients. By estimating the MA model and obtaining the residuals and estimated coefficients from it, we can test the time reversibility according to necessary and sufficient condition for time reversibility of stationary linear processes.

The estimated model $X_n = \sum_{i=0}^{n_0} b_i Z_{n-i}$ has only one of the following situations:

I. The process is Gaussian process.

II. It is non Gaussian process, $\{Z_n : n \in \mathbb{Z}\}$ has symmetrical distribution around zero and $b_n = -b_{n_0-n}$.

III. 3- The process is non-Gaussian process and $\{Z_n : n \in \mathbb{Z}\}$ has non symmetrical distribution around zero and $b_n = b_{n_0-n}$.

IV. The model is not time reversible.

Remark 3.1. *If the process is linear, this procedure will give an exact test for time reversibility, otherwise, the procedure starts with substitution a MA representation as a local approximation to the unknown stationary model and then tests the reversibility of approximation model.*

Note that, we fit the best MA representation by known criterion. Using the fact that under assumption of Theorem 2.1, the representation $X_n = \sum_{j \in Z} b_j Z_{n-j}$ *for non Gaussian stationary processes is unique, if we conclude reversibility of MA representation of the process* $\{X_n : n \in \mathbb{Z}\}$*, the reversibility of the process won't be rejected (Cheng, 1992).*

The first step is fitting an invertible MA model to the data. If the time series is Gaussian, then it is reversible and it is not necessary to check the symmetrically distribution of residuals and relationship between coefficients. If the time series is non-Gaussian, the next step is testing whether the distribution of the residuals is symmetric or not, and then checking the relationship between coefficients by standard tests.

3.1 Testing Gaussianity

There are several methods to test Gaussianity of time series. These methods include histogram plot, kurtosis test and hypothesis testing using cumulants and bispectrum of the available sequence and in base of skewness and kurtosis.

Hinich (1982) has given a test for Gaussianity of a stationary time series using cumulant and bispectrum of data. Hinich's test examines whether the imaginary part of the estimated bispectrum is equal to zero or not. It should be pointed out that a zero bispectrum is not a proof of Gaussianity. Because if a random process is symmetrically distributed, its order cumulant is zero. So, other tests such as the kurtosis test should be employed.

Also Bai and Ng (2005) introduced a test for Gaussianity for time series based on skewness and kurtosis that data is stationary up to eighth order. They test the Gaussianity test of real data and examples by testing whether the bispectrum of the given data is non zero or not. If the bispectrum is zero, then the stochastic process could be Gaussian but not necessarily true. While the histogram provides a visual impression of the shape of distribution of the measurements, the kurtosis measures the degree of peakness or flatness of a distribution. Here, to check Gaussianity of the data, we apply the kurtosis test.

Also Hinich and Rothman (1998) introduced a frequency domain test of time reversibility for third order stationary time series based on the bispectrum related to Hinich's (1982) and Gaussianity test. In particular, it checks whether the breakdown of Gaussianity is due to time irreversibility or not.

3.2 Checking Distributions and Coefficients

When $X_n = \sum_{i=0}^{n_0} b_j Z_{n-j}$, testing the first part of the conditions (II) and (III) are well known. For testing the second parts which is the relationship between coefficients, if n_0 is even we should test the hypotheses (3.1) versus that the equations do not satisfy which is not hard to check.

$$\begin{bmatrix} 1 & 0 & & \cdots & & 0 & \pm 1 \\ 0 & 1 & & & & \pm 1 & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & 1 & 0 & \pm 1 & \cdots & 0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n_0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{3.1}$$

When n_0 is odd, a similar equations should satisfy.
Many authors distinct the concept of time reversibility and being stationary and notions of linearity or nonlinearity, but note that we use this fact that every stationary time series can be approximated by a linear model. If the processes is linear, this procedure is an exact test of reversibility. If the time series is nonlinear, then the procedure contains approximating since we substitute an MA representation as a local approximation to the unknown stationary model.

4. Simulation Study

In this Section, as an illustration, we investigate the finite sample performance of the procedure by simulation.
First, we generate 1000 iteration of a known reversible or irreversible time series with sample size 100 and fit a MA model to the simulated data. The optimal value of the order of MA will be obtained by use of the Akaike Information Criterion (AIC), or something similar, such as the Schwartz Information Criterion (SIC). Then we test the Gaussianity of time series. If the process is Gaussian, the time series is reversible, otherwise, we will test the relationship between coefficients in according to result of investigating for symmetric or non symmetric distributions around zero of residuals. We report the mean of p-values in the tables. In all examples, the results of the procedure confirm the theoretical methods. The details of examples are available by authors.

Remark 4.1. *In some of next examples and real data, the fitted models for non-Gaussian time series $\{X_n : n \in \mathbb{Z}\}$ will be $X_n = Z_n + \theta Z_{n-1}$. If we consider the time reversibility condition for distribution and coefficients of innovations, θ must be -1 for symmetric distribution about* *zero and +1 for non symmetric distribution and we know that these MA model are not invertible models. So the non-Gaussian MA(1) models are irreversible and we don't check the relationship between coefficients in base of distribution of residuals.*

Example 4.1. In a 1000 simulation of reversible MA model of size 100 as $X_n = Z_n - Z_{n-1} + Z_{n-2}$, where Z_n's are iid with known distribution mentioned in the following table, we fit a MA model and test the Gaussianity. For two cases $Gamma(0.8, 1)$ and $Beta(0.5, 5)$ that p-values don't accept the Gaussianity, we test the conditions (I) and (II) of Theorem(2.1) in base of the distribution of residuals. The results are given in Table 1.

Table 1: The result of Example 4.1

Distribution	n_0	Mean of p-value(Gaussianity)	Mean of p-value(II, III)
$Exp\,(2)$	2	0.2288	–
$Gamma\,(0.8, 1)$	2	0.01887	> 0.995
$T\,(3)$	2	0.6312	–
$\chi^2\,(3)$	2	0.1812	–
$f\,(2, 5)$	2	0.1694	–
$Beta\,(0.5, 5)$	2	0.01380	> 0.995

Following the mean of p-values in Table 1, the results confirm the reversibility of the model.

Example 4.2. Our simulation has been based on 1000 iteration of the irreversible MA(1) with sample size 100 as $X_n = Z_n - Z_{n-1}$, where Z_n's are iid with known distribution. The results are collected in Table 2.

Table 2: The result of Example 4.2

Distribution	n_0	Mean of p-value(Gaussianity)	Mean of p-values (II, III)
$Exp\,(2)$	3	0.01141	< 0.005
$Gamma\,(2, 3)$	5	0.01259	< 0.005
$t\,(3)$	2	3.7×10^{-11}	< 0.005
$\varkappa^2\,(3)$	1	2.478×10^{-4}	–
$f\,(2, 5)$	1	2.2×10^{-16}	–

As we see from Table 2, the Gaussianity hypothesis is rejected for all innovation distributions. So as we noted in Remark 4.1, we don't check the conditions (I) and (II) for them and we conclude that they are irreversible. The last column tests the relationship between coefficients in according to type of distributions of residuals in non-Gaussian time series. Therefore, for all of cases, we conclude irreversibility of this model.

Example 4.3. Consider the nonlinear time reversible random coefficient as $X_n = Z_n + B_n Z_{n-1}$, where $Z_n \overset{iid}{\sim} N\left(0, \sigma^2\right)$ and $B_n \overset{iid}{\sim} Beta\left(\alpha, \beta\right)$ are independent. Since $\{B_n\}$ and $\{Z_n\}$ are iid and regarding the fact that they are independent from each other, we have

$$\begin{pmatrix} X_{\tau-n} \\ X_{\tau-n-1} \\ . \\ . \\ . \\ X_{\tau-n-k} \end{pmatrix} \overset{d}{=} \begin{pmatrix} 1 & B_{\tau-n} & 0 & ... & 0 & 0 \\ 0 & 1 & B_{\tau-n-1} & & 0 & 0 \\ . & . & . & & . & . \\ . & . & . & & . & . \\ . & . & . & & . & . \\ 0 & 0 & 0 & & 1 & B_{\tau-n-k} \end{pmatrix} \begin{pmatrix} Z_{\tau-n} \\ Z_{\tau-n-1} \\ . \\ . \\ . \\ Z_{\tau-n-k-1} \end{pmatrix}$$

$$\overset{d}{=} \begin{pmatrix} 1 & B_n & 0 & ... & 0 & 0 \\ 0 & 1 & B_{n+1} & & 0 & 0 \\ . & . & . & & . & . \\ . & . & . & & . & . \\ . & . & . & & . & . \\ 0 & 0 & 0 & & 1 & B_{n-k} \end{pmatrix} \begin{pmatrix} Z_n \\ Z_{n+1} \\ . \\ . \\ . \\ Z_{n+k+1} \end{pmatrix}$$

$$\overset{d}{=} \begin{pmatrix} X_n \\ X_{n+1} \\ . \\ . \\ . \\ X_{n+k} \end{pmatrix}$$

Therefore, this process is time reversible. We simulate this process with replication 1000, n=100 and different α, β and k. The results are shown in Table 3.

Table 3: The result of Example 4.3

α	σ^2	k	n_0	Mean of p-value(Gaussianity)
0.1	9	10	1	0.5552
0.5	9	10	1	0.7766
0.9	4	10	1	0.8577
0.5	4	5	1	0.5388

For all of the 1000 replications, this process has the same structure as the usual Gaussian MA process, except that here the coefficient $\{B_n\}$, is a random variable rather than a constant.

Example 4.4. Bilinear processes are popular nonlinear models that can capture a wide range of nonlinear behavior. In the bilinear case, we specify the data generating process as $X_n = \alpha X_{n-1} + \beta X_{n-1} Z_{n-1} + Z_n$ where $\{Z_n : n \in \mathbb{Z}\}$ is a sequence of independently and identically distributed $N(0, 1)$ random variables. . This model was studied by Ramsey and Rothman (1996) and they showed that this process is time irreversible. We simulate 1000 realization of this model for different α and β and for $n = 100$. We fit a MA model to the simulated data and test the Gaussianity. Then we test the conditions (II) and (III) to see whether the residuals are symmetric or not in non-Gaussian MA models. The results are appeared in Table 4.

Table 4: The result of Example 4.4

α	β	n_0	Mean of p-value(Gaussianity)	Mean of p-value (II, III)
0.2	5	3	$< 2.2 \times 10^{-16}$	< 0.005
0.2	2	15	$< 2.2 \times 10^{-16}$	< 0.005
2	0.5	13	$< 2.2 \times 10^{-16}$	< 0.005
0.5	0.5	2	3.818×10^{-6}	< 0.005

As we see, this process is non-Gaussian and the conditions (I) and (II) for relationship between coefficients according to distribution of innovation are not accepted. So the result of the procedure confirms the theoretical subject.

5. Real Data

To use our procedure in real data sets , we investigate the time reversibility of Backus and Kehoe international data sets. We examine twenty four series, four indicators for six different countries. The four indicators chosen are real output, investment, price level and money supply that were selected for Australia, Canada, Italy, Sweden, The United Kingdom (UK) and The United States (US). Definitions, data sources and data from each series can be found in the appendix of Backus and Kehoe (1992).

For all of the twenty four series, we analyze the growth rates by fitting a MA model to the data, testing the Gaussianity, obtaining the estimation of the innovations and coefficients, and then testing wheater the innovation distribution are symmetric and testing relationship between coefficients in non-Gaussian MA models. The results of the procedure are presented in Table 5 and the details of the results are available by authors.

Table 5: The result of reversibility test for Backus and Kehoe international data sets.

Country	Series	n_0	Mean of p-value(Gaussianity),(II, III)
Australia	Real Output$(1861-1985)$	$-$	0.291 , $-$
	Investment$(1861-1985)$	1	$< 2.2\times 10^{-16}$, $-$
	Price Level$(1861-1985)$	4	4.421×10^{-3} , < 0.005
	Money Supply$(1870-1970)$	1	1.498×10^{-4} , $-$
Canada	Real Output$(1870-1983)$	$-$	0.0791, $-$
	Investment$(1870-1983)$	1	3.7×10^{-2}, $-$
	Price Level$(1870-1983)$	3	3.51×10^{-4} , < 0.005
	Money Supply$(1870-1975)$	$-$	0.3081 , $-$
Italy	Real Output$(1861-1985)$	1	5.844×10^{-13} , $-$
	Investment$(1861-1985)$	1	2.006×10^{-13} , $-$
	Price Level$(1861-1985)$	4	$< 2.2\times 10^{-16}$, < 0.005
	Money Supply$(1870-1975)$	8	1.076×10^{-9} , < 0.005
Sweden	Real Output$(1861-1986)$	11	1.491×10^{-5} , < 0.005
	Investment$(1861-1986)$	12	1.182×10^{-7} , < 0.005
	Price Level$(1861-1986)$	3	4.05×10^{-8} , < 0.005
	Money Supply$(1871-1975)$	7	5.149×10^{-7} , < 0.005
UK	Real Output$(1870-1986)$	4	7.483×10^{-6} , < 0.005
	Investment$(1870-1986)$	1	4.718×10^{-16} , $-$
	Price Level$(1870-1986)$	3	1.752×10^{-6} , < 0.005
	Money Supply$(1871-1975)$	3	3.754×10^{-8} , < 0.005
US	Real Output$(1869-1983)$	$-$	0.2178 , $-$
	Investment$(1889-1988)$	5	8.231×10^{-4} , < 0.005
	Price Level$(1869-1983)$	3	5.321×10^{-3} , < 0.005
	Money Supply$(1867-1975)$	2	0.03256 , < 0.005

Ramsey and Rothman (1996) analyzed this data set and concluded as below:

1- Fifteen of the twenty four series were nonlinear irreversible.

2- Four series are seemingly linear with non-Gaussian innovation. These four series are Australian price level and money supply, Swedish real output and US money supply.

3- Two series are anomalies, the Canadian and Italian money supply series, in that they have acceptance with the raw data, but rejection with the ARMA residuals.

4- Three series of growth rates from the Backus and Kehoe data set were time reversible of order 3 and degree 5: Australian and US real output and Canadian investment. They also tested the derivatives of each growth rate series for evidence of transversal asymmetry. Canadian investment and US real output were rejected for transval symmetry at p-values of 0.065 and 0.025, respectively. Only the Australian real output growth real series and its derivative appeared to be time reversible.

We conclude as below:

1- Four of twenty four series are Gaussian and it is appeared that they are time reversible. These four series, Canadian money supply, Canadian real output, Australian real output and US real output are Gaussian at significant level lower than 0.005 that two of them consistent to the results of Ramsey and Rothman (1996).

2- Six of twenty four series are fitted by MA(1) and non-Gaussian and their coefficients are not. So the reversibility hypothesis is rejected. These six series are Australian, Canadian, Italian and UK Investment, Australian money supply and Italian real output.

3- Fourteen of them are fitted by MA , and non-Gaussian, but the coefficients and innovations don't have the necessary and sufficient conditions of reversible non-Gaussian linear model.

6. Summary

In this paper, we propose a simple procedure for analysis of time reversibility in base of a necessary and sufficient condition for time reversibility of linear models. In contrast, RRT, Hinich and Rothman (1998), CCKT and Psaradakis (2008) are in base of a necessary condition for time reversibility. A useful feature of this procedure is that it doesn't require existence of moments of higher than two and we only fit an approximate linear model MA to the stationary data.

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