Journal of Mathematical Extension
Vol. 15, No. 2, (2021) (12)1-11
URL: https://doi.org/10.30495/JME.2021.1403
ISSN: 1735-8299
Original Research Paper

On (n, m)-Jordan Homomorphisms on Algebras

M. Neghabi
Islamic Azad University of South Tehran

A. Bodaghi*
Islamic Azad University of Garmsar

A. Zivari-Kazempour
Ayatollah Borujerdi University

Abstract. In this paper, it is shown that every (n, m)-Jordan homomorphism between two commutative algebras is an (n, m)-homomorphism. For the non-commutative case, it is proved that every surjective $(2, m)$-Jordan homomorphism from an algebra $\mathcal{A}$ into a semiprime commutative algebra $\mathcal{B}$ is a $(2, m)$-homomorphism.

AMS Subject Classification: Primary 47B48; Secondary 46L05, 46H25
Keywords and Phrases: n-homomorphism, n-Jordan homomorphism, (n, m)-Jordan homomorphism, semisimple.

1 Introduction and Preliminaries

Let $\mathcal{A}$ and $\mathcal{B}$ be complex algebras and $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ be a linear map. Then, φ is called an *n-homomorphism* if for all $a_1, a_2, \dots, a_n \in \mathcal{A}$,

$$\varphi(a_1 a_2 \cdots a_n) = \varphi(a_1)\varphi(a_2) \cdots \varphi(a_n).$$

Received: September 2019; Accepted: February 2020
*Corresponding Author

The concept of an n-homomorphism was introduced and studied for complex algebras in [5] and [7]. Moreover, φ is called an *n-Jordan homomorphism* if

$$\varphi(a^n) = \varphi(a)^n, \qquad (a \in \mathcal{A}).$$

This notion was introduced by Herstein in [8]. A 2-homomorphism (resp. 2-Jordan homomorphism) is called simply a homomorphism (resp. Jordan homomorphism). Obviously, every n-homomorphism is an n-Jordan homomorphism, but the converse is not valid in general. It is shown in [9] that some Jordan homomorphism on the polynomial rings can not be homomorphism. For Jordan homomorphisms on Banach algebras, Zelazko in [13] presented the following result (see also [11] for an alternative proof).

Theorem 1.1. *Suppose that $\mathcal{A}$ is a Banach algebra, which need not be commutative, and suppose that $\mathcal{B}$ is a semisimple commutative Banach algebra. Then, each Jordan homomorphism $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ is a homomorphism.*

Next, the above result has been proved by the third author in [14] for 3-Jordan homomorphism with the extra condition that the Banach algebra $\mathcal{A}$ is unital. Recently, the second author and İnceboz extended Theorem 1.1 for $n \in \{3, 4\}$ in [4] without the condition that $\mathcal{A}$ is unital by considering an alternative condition on n-Jordan homomorphisms. For the general case, G. An [1] extended the mentioned results for all $n \in \mathbb{N}$ and showed that for a unital ring $\mathcal{A}$ and a ring $\mathcal{B}$ with char($\mathcal{B}$)$> n$, every n-Jordan homomorphism from $\mathcal{A}$ into $\mathcal{B}$ is an n-homomorphism provided that every Jordan homomorphism from $\mathcal{A}$ into $\mathcal{B}$ is a homomorphism. For different proofs of the case that $\mathcal{A}$ and $\mathcal{B}$ are commutative algebras, we refer to [3], [6] and [10]. We also refer to [12] for characterization of mixed n-Jordan homomorphisms and pseudo n-Jordan homomorphisms.

In this paper, we show that every (n, m)-Jordan homomorphism $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ between two commutative algebras is an (n, m)-homomorphism. In addition, we prove that every unital $(n+1, m)$-Jordan homomorphism $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ is an (n, m)-Jordan homomorphism. As a result, every unital $(2, m)$-Jordan homomorphism $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ is a $(2, m^{n-1})$-Jordan homomorphism, for $n \geq 2$. Furthermore, for an algebra $\mathcal{A}$ and a semiprime commutative algebra $\mathcal{B}$ we show that each surjective $(2,m)$-Jordan homomorphism $\varphi:\mathcal{A}\longrightarrow\mathcal{B}$ can be a $(2,m)$-homomorphism.

2 (n,m)-Jordan homomorphisms

Let $m\in\mathbb{Z}\setminus\{0\}$ be fixed, $\mathcal{A}$ and $\mathcal{B}$ be complex algebras and $\varphi:\mathcal{A}\longrightarrow\mathcal{B}$ be a linear map. Then, φ is called an (n,m)-*homomorphism* if for all $a_1,a_2,\ldots,a_n\in\mathcal{A}$,

$$\varphi(a_1a_2\cdots a_n)=m\varphi(a_1)\varphi(a_2)\cdots\varphi(a_n).$$

Moreover, the mapping φ is said to be an (n,m)-*Jordan homomorphism* if

$$\varphi(a^n)=m\varphi(a)^n,\qquad a\in\mathcal{A}.$$

The concept of (n,m)-Jordan homomorphism was recently introduced by the third author in [15]. Clearly, $(n,1)$-homomorphism and $(n,1)$-Jordan homomorphism coincide with the classical definitions of n-homomorphism and n-Jordan homomorphism, respectively.

Note that every n-Jordan homomorphism is not necessarily (n,m)-Jordan homomorphism for $m\neq 1$ and vice versa in general. For example, define $\varphi:\mathbb{R}\longrightarrow\mathbb{R}$ by $\varphi(x)=\frac{1}{2}x$. Then, φ is not n-Jordan homomorphism, but for $m=2^{(n-1)}$ it is (n,m)-Jordan homomorphism. We bring the next example which is presented in [15].

Example 2.1. Let

$$\mathcal{A}=\left\{\begin{bmatrix}X & 0\\ 0 & Y\end{bmatrix}:\quad X,Y\in M_2(\mathbb{C})\right\},$$

and consider the mapping $\varphi:\mathcal{A}\longrightarrow\mathcal{A}$ defined through

$$\varphi\left(\begin{bmatrix}X & 0\\ 0 & Y\end{bmatrix}\right)=\frac{1}{k}\begin{bmatrix}X & 0\\ 0 & Y^T\end{bmatrix},$$

for each $k\in\mathbb{N}$, where Y^T is the transpose of matrix Y. For all $U\in\mathcal{A}$, we have

$$\varphi(U^n)-k^{(n-1)}\varphi(U)^n=\frac{1}{k}\begin{bmatrix}X^n & 0\\ 0 & (Y^n)^T\end{bmatrix}-k^{(n-1)}\frac{1}{k^n}\begin{bmatrix}X^n & 0\\ 0 & (Y^T)^n\end{bmatrix}=0.$$

Thus, φ is (n,m)-Jordan homomorphism for $m = k^{(n-1)}$ but not (n,m)-homomorphism.

In the sequel, $\left(\begin{array}{c} n \\ k \end{array}\right)$ is the binomial coefficient defined for all $n, k \in \mathbb{N}$ with $n \geq k$ by $n!/(k!(n-k)!)$.

To achieve some main results in this section, we need the following lemma which is proved in [3, Lemma 2.1].

Lemma 2.2. *If* $x_1, x_2, \cdots, x_n \in \mathbb{R}$, *then*

$$det\left(\begin{bmatrix} x_1 & x_1^2 & \cdots & x_1^n \\ x_2 & x_2^2 & \cdots & x_2^n \\ \vdots & \vdots & \ddots & \vdots \\ x_n & x_n^2 & \cdots & x_n^n \end{bmatrix}\right) = (-1)^{\frac{n(n-1)}{2}} \prod_{k=1}^{n} x_k \prod_{i<j} (x_i - x_j).$$

For $m = 1$, the next result is Theorem 2.7 of [16]. Here, we generalize it for all $m \in \mathbb{Z} \setminus \{0\}$.

Theorem 2.3. *Every unital* $(n+1, m)$*-Jordan homomorphism* $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ *is an* (n, m)*-Jordan homomorphism.*

Proof. We firstly have

$$\varphi\left((a + le)^{n+1}\right) = m\left(\varphi\left(a + le\right)\right)^{n+1} \tag{1}$$

for all $a \in \mathcal{A}$, where l is an integer with $1 \leq l \leq n$ and e is the identity of $\mathcal{A}$. It follows from equality (1) and assumption that

$$\sum_{i=1}^{n} l^{n+1-i} \left(\begin{array}{c} n+1 \\ i \end{array}\right) \left[\varphi(a^i) - m\varphi(a)^i\right] = 0, \qquad (1 \leq l \leq n), \tag{2}$$

for all $a \in \mathcal{A}$. We can represent the equalities in (2) as the matrix form

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 2^n & 2^{n-1} & \cdots & 2 \\ 3^n & 3^{n-1} & \cdots & 3 \\ \vdots & \vdots & \ddots & \vdots \\ n^n & n^{n-1} & \cdots & n \end{bmatrix} \begin{bmatrix} \Gamma_1(a) \\ \Gamma_2(a) \\ \Gamma_3(a) \\ \vdots \\ \Gamma_n(a) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

where $\Gamma_i(a) = \begin{pmatrix} n+1 \\ i \end{pmatrix} \left[\varphi(a^i) - m\varphi(a)^i\right]$ for all $1 \le i \le n$ and $a \in \mathcal{A}$. Since the above square matrix is invertible (Lemma 2.2), $\Gamma_i(a)$ should be zero for all $1 \le i \le n$ and all $a \in \mathcal{A}$. In particular, $\Gamma_n(a) = 0$. This means that φ is a (n,m)-Jordan homomorphism. $\square$

The proof technique of the following theorem is taken from [3, Theorem 2.2]. We include some parts for the sake of completeness.

Theorem 2.4. *Every (n,m)-Jordan homomorphism between two commutative algebras $\mathcal{A}$ and $\mathcal{B}$ is an (n,m)-homomorphism.*

Proof. Assume that $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ is an (n,m)-Jordan homomorphism. By assumption, we have $\varphi\left((x+ky)^n\right) = m(\varphi(x+ky))^n$ for all $x, y \in \mathcal{A}$, where k is an integer with $2 \le k \le n$. Thus

$$\varphi\left(\sum_{j=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} k^j x^{n-j} y^j\right) = \sum_{j=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} k^j m\varphi(x)^{n-j}\varphi(y)^j$$

for all $x, y \in \mathcal{A}$. Similar to the proof of [3, Theorem 2.2], one can show that $F_j(x,y) = 0$ for all $1 \le j \le n-1$ and all $x, y \in \mathcal{A}$, where $F_j(x,y) = \begin{pmatrix} n \\ j \end{pmatrix} \left[\varphi\left(x^{n-j}y^j\right) - m(\varphi(x))^{n-j}(\varphi(y))^j\right]$. In particular, $F_{n-1}(x,y) = 0$ for all $x, y \in \mathcal{A}$. Hence,

$$\varphi\left(xy^{n-1}\right) = m\varphi(x)(\varphi(y))^{n-1}$$

for all $x, y \in \mathcal{A}$. Moreover, similar to the second part of mentioned proof, one can conclude that

$$\varphi\left(x_1 x_2 \cdots x_l x_{l+1}^{n-l}\right) = m\varphi(x_1)\varphi(x_2)\cdots\varphi(x_l)\varphi\left(x_{l+1}^{n-l}\right)$$

for all $1 \le l \le n-1$ and all $x_1, x_2, \ldots, x_l, x_{l+1} \in \mathcal{A}$. This means that φ is an (n,m)-homomorphism. $\square$

Proposition 2.5. *Every $(2,m)$-Jordan homomorphism φ between algebras $\mathcal{A}$ and $\mathcal{B}$ is $\left(n, m^{n-1}\right)$-Jordan homomorphism, for $n \ge 2$.*

Proof. We argue by induction on n. In other words, we suppose that φ is $\left(k, m^{k-1}\right)$-Jordan homomorphism, for $k \ge 2$ and then prove that it is

$\left(k+1, m^k\right)$-Jordan homomorphism. By assumption, we have $\varphi\left(a^2\right) = m\varphi(a)^2$, for all $a \in \mathcal{A}$. Replacing a by $a+b$, we get

$$\varphi(ab+ba) = m\big(\varphi(a)\varphi(b) + \varphi(b)\varphi(a)\big). \tag{3}$$

Interchanging b by a^2 in (3), gives

$$\varphi\left(a^3\right) = m^2\varphi(a)^3$$

for all $a \in \mathcal{A}$, and so φ is $(3, m^2)$-Jordan homomorphism. Substituting b by a^k in (3), we find

$$2\varphi\left(a^{k+1}\right) = m\left(\varphi(a)\varphi\left(a^k\right) + \varphi\left(a^k\right)\varphi(a)\right), \quad (a \in \mathcal{A}). \tag{4}$$

On the other hand, we have

$$\varphi\left(a^k\right) = m^{k-1}\varphi(a)^k \tag{5}$$

for all $a \in \mathcal{A}$. By (4) and (5), we obtain $\varphi(a^{k+1}) = m^k\varphi(a)^{k+1}$. □

The next corollary is a direct consequence of Theorem 2.3 and Proposition 2.5 and so we include it without proof.

Corollary 2.6. *Every unital $(2, m)$-Jordan homomorphism $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ is a $\left(2, m^{n-1}\right)$-Jordan homomorphism, for $n \geq 2$.*

The upcoming lemma help us to show that each surjective $(2, m)$-Jordan homomorphism is a $(2, m)$-homomorphism under some mild conditions on algebras.

Lemma 2.7. *Suppose that $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ is a $(2, m)$-Jordan homomorphism. Then*

(1) $\varphi(aba) = m^2\varphi(a)\varphi(b)\varphi(a)$;

(2) $\varphi(abc + cba) = m^2\big(\varphi(a)\varphi(b)\varphi(c) + \varphi(c)\varphi(b)\varphi(a)\big)$,

for all $a, b, c \in \mathcal{A}$.

Proof. Let φ be a $(2, m)$-Jordan homomorphism. Then, $\varphi(a^2) = m\varphi(a)^2$, for all $a \in \mathcal{A}$. Replacing a by $a + b$, we get

$$\varphi(ab + ba) = m\big(\varphi(a)\varphi(b) + \varphi(b)\varphi(a)\big), \tag{6}$$

for all $a, b \in \mathcal{A}$. Interchanging a by a^2 in (6), we have

$$\varphi(a^2b + ba^2) = m^2\left(\varphi(a)^2\varphi(b) + \varphi(b)\varphi(a)^2\right), \tag{7}$$

for all $a, b \in \mathcal{A}$. Once more, by switching b into $ab+ba$ in (6), we obtain

$$\varphi(a(ab + ba) + (ab + ba)a) = m\big(\varphi(a)\varphi(ab + ba) + \varphi(ab + ba)\varphi(a)\big),$$

and hence by (6), we arrive at

$$\varphi\left(a^2b + 2aba + ba^2\right) = m^2\left(\varphi(a)^2\varphi(b) + \varphi(b)\varphi(a)^2\right)+2m^2\varphi(a)\varphi(b)\varphi(a), \tag{8}$$

for all $a, b \in \mathcal{A}$. Subtraction (7) from (8), gives

$$\varphi(aba) = m^2\varphi(a)\varphi(b)\varphi(a). \tag{9}$$

Therefore, part (1) is shown. Now, let $a, b, c \in \mathcal{A}$ be arbitrary. We have

$$abc + cba = (a + c)b(a + c) - aba - cbc,$$

and so by (9), we find

$$\begin{aligned}
&\varphi(abc + cba)\\
&\quad= \varphi\big((a + c)b(a + c)\big) - \varphi(aba) - \varphi(cbc)\\
&\quad= m^2\big(\varphi(a + c)\varphi(b)\varphi(a + c) - \varphi(a)\varphi(b)\varphi(a) - \varphi(c)\varphi(b)\varphi(c)\big)\\
&\quad= m^2\big([\varphi(a)\varphi(b) + \varphi(c)\varphi(b)]\varphi(a + c) - \varphi(a)\varphi(b)\varphi(a) - \varphi(c)\varphi(b)\varphi(c)\big)\\
&\quad= m^2\big(\varphi(a)\varphi(b)\varphi(a) + \varphi(a)\varphi(b)\varphi(c) + \varphi(c)\varphi(b)\varphi(a) + \varphi(c)\varphi(b)\varphi(c)\\
&\qquad\quad - \varphi(a)\varphi(b)\varphi(a) - \varphi(c)\varphi(b)\varphi(c)\big)\\
&\quad= m^2\big(\varphi(a)\varphi(b)\varphi(c) + \varphi(c)\varphi(b)\varphi(a)\big).
\end{aligned}$$

Thus, $\varphi(abc + cba) = m^2\big(\varphi(a)\varphi(b)\varphi(c) + \varphi(c)\varphi(b)\varphi(a)\big)$, for all $a, b, c \in \mathcal{A}$, which completes the proof. $\square$

For a commutative ring $\mathcal{R}$, a proper ideal $\mathcal{I}$ is a *semiprime* ideal if $\mathcal{I}$ satisfies either of the following equivalent conditions:

• If x^k is in $\mathcal{I}$ for some positive integer k and element x of $\mathcal{R}$, then x is in $\mathcal{I}$.

• If y is in $\mathcal{R}$ but not in $\mathcal{I}$, all positive integer powers of y are not in $\mathcal{I}$.

A ring $\mathcal{R}$ is called *semiprime* if the zero ideal is a semiprime ideal. A left ideal $\mathcal{I}$ of an algebra $\mathcal{A}$ is a modular left ideal if there exists $u \in \mathcal{A}$ such that $\mathcal{A}(e_{\mathcal{A}} - u) \subseteq \mathcal{I}$, where $\mathcal{A}(e_{\mathcal{A}} - u) = \{x - xu : x \in \mathcal{A}\}$. The Jacobson radical Rad $(\mathcal{A})$ of $\mathcal{A}$ is the intersection of all maximal modular left ideals of $\mathcal{A}$. An algebra $\mathcal{A}$ is called *semisimple* whenever its Jacobson radical Rad $(\mathcal{A})$ is trivial.

It is known that every semisimple algebra is semiprime [2], but the converse is not generally true. For a semiprime algebra $\mathcal{B}$, the next result characterizes $(2, m)$-Jordan homomorphisms.

Theorem 2.8. *Let $\mathcal{A}$ be an algebra, and $\mathcal{B}$ be a semiprime commutative algebra. Then, each surjective $(2, m)$-Jordan homomorphism $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ is a $(2, m)$-homomorphism.*

Proof. Suppose that $t = \varphi(ab) - m\varphi(a)\varphi(b)$. By assumption, for any $y \in \mathcal{B}$ there exists $x \in \mathcal{A}$ such that $y = \varphi(x)$. By Lemma 2.7, we have

$$\begin{aligned}
tyt &= \varphi(ab)\varphi(x)\varphi(ab) + m^2\varphi(a)\varphi(b)\varphi(x)\varphi(a)\varphi(b) \\
&\quad - 2m\varphi(ab)\varphi(x)\varphi(a)\varphi(b) \\
&= \frac{1}{m^2}\varphi(abxab) + \varphi(a)\varphi(bxb)\varphi(a) - 2m\varphi(ab)\varphi(x)\varphi(a)\varphi(b) \\
&= \frac{1}{m^2}\varphi(abxab) + \frac{1}{m^2}\varphi(abxba) - 2m\varphi(ab)\varphi(x)\varphi(a)\varphi(b). \qquad (10)
\end{aligned}$$

Similarly,

$$tyt = \frac{1}{m^2}\varphi(abxab) + \frac{1}{m^2}\varphi(baxab) - 2m\varphi(ab)\varphi(x)\varphi(a)\varphi(b). \qquad (11)$$

Plugging (10) into (11), we conclude that

$$\begin{aligned}2tyt &= tyt + tyt \\ &= \frac{2}{m^2}\varphi(abxab) + \frac{1}{m^2}\varphi(abxba) + \frac{1}{m^2}\varphi(baxab) \\ &\quad - m[(\varphi(a)\varphi(b) + \varphi(b)\varphi(a))\varphi(x)\varphi(ab) \\ &\quad + \varphi(ab)\varphi(x)(\varphi(a)\varphi(b) + \varphi(b)\varphi(a))] \\ &= \frac{2}{m^2}\varphi(abxab) + \frac{1}{m^2}\varphi(abxba) + \frac{1}{m^2}\varphi(baxab) \\ &\quad - \frac{1}{m^2}\varphi[(ab+ba)x(ab) + (ab)x(ab+ba)] \\ &= 0.\end{aligned}$$

Therefore, $tyt = 0$. Since $\mathcal{B}$ is semiprime, we have $t = 0$ and so $\varphi(ab) = m\varphi(a)\varphi(b)$ which finishes the proof. □

Corollary 2.9. *Let $\mathcal{A}$ be a unital algebra, and $\mathcal{B}$ be a semiprime commutative algebra. If $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ is a surjective $\left(3, m^2\right)$-Jordan homomorphism, then φ is either $(2, m)$-homomorphism or $(2, -m)$-homomorphism.*

Proof. The result follows immediately from Theorem 2.8 and Lemma 2.4 of [15]. □

As a consequence of the preceding corollary, we have the next result.

Corollary 2.10. *Suppose that $\mathcal{A}$ is a unital algebra and $\mathcal{B}$ is a semiprime commutative algebra. Then, each surjective $\left(3, m^2\right)$-Jordan homomorphism $\varphi : \mathcal{A} \longrightarrow \mathcal{B}$ is a $\left(3, m^2\right)$-homomorphism.*

Acknowledgments

The authors would like to thank the reviewers for careful reading of the paper and giving some useful comments.

References

[1] G. An, Characterization of n-Jordan homomorphism, *Linear Multilinear Algebra*, 66 (4) (2018), 671–680.

[2] F. F. Bonsall and J. Duncan, *Complete normed algebra*, Springer-Verlag, New York, (1973).

[3] A. Bodaghi and H. İnceboz, n-Jordan homomorphisms on commutative algebras, *Acta. Math. Univ. Comenianae.*, 87 (1), (2018), 141–146 .

[4] A. Bodaghi and H. İnceboz, Extension of Zelazko's theorem to n-Jordan homomorphisms, *Adv. Pure Appl. Math.*, 10 (2019), no. 2, 165-170.

[5] J. Bračič and M. S. Moslehian, On automatic continuity of 3-homomorphisms on Banach algebras, *Bull. Malaysian Math. Sci. Soc.*, 30 (2), (2007), 195–200.

[6] E. Gselmann, On approximate n-Jordan homomorphisms, *Annales Math. Silesianae.*, 28 (2014), 47–58.

[7] Sh. Hejazian, M. Mirzavaziri and M. S. Moslehian, n-homomorphisms, *Bull. Iranian Math. Soc.*, 31 (1), (2005), 13–23.

[8] I. N. Herstein, Jordan homomorphisms, *Trans. Amer. Math. Soc.*, 81 (1), (1956), 331–341.

[9] N. Jacobson and C. E. Rickart, Jordan homomorphisms of rings, *Trans. Amer. Math. Soc.*, 69 (3), (1950), 479–502.

[10] Y. H. Lee, Stability of n-Jordan homomorphisms from a normed algebra to a Banach algebra, *Abst. Appli. Anal.*, 2013, Article ID 691025, (2013), 5 pages.

[11] T. Miura, S. E. Takahasi, and G. Hirasawa, Hyers-Ulam-Rassias stability of Jordan homomorphisms on Banach algebras, *J. Ineq. Appl.*, 2005 (4), (2005), 435–441.

[12] M. Neghabi, A. Bodaghi and A. Zivari-Kazempour, Characterization of mixed n-Jordan homomorphisms and pseudo n-Jordan homomorphisms, *Filomat*, 34 (2020), no. 6, 1989–2002.

[13] W. Zelazko, A characterization of multiplicative linear functionals in complex Banach algebras, *Studia Math.*, 30 (1968), 83–85.

[14] A. Zivari-Kazempour, A characterization of 3-Jordan homomorphism on Banach algebras, *Bull. Aust. Math. Soc.*, 93 (2), (2016), 301–306.

[15] A. Zivari-Kazempour, Characterization of (n, m)-Jordan homomorphisms, *Bull. TICMI*, 23 (1), (2019), 25–33.

[16] A. Zivari-Kazempour, Automatic continuity of n-Jordan homomorphisms on Banach algebras, *Commun. Korean Math. Soc.*, 33 (1), (2018), 165–170.

Masoumeh Neghabi
Ph.D Student of Mathematics
Department of Mathematics
South Tehran Branch, Islamic Azad University
Tehran, Iran
E-mail: m.neghabi114@gmail.com

Abasalt Bodaghi
Professor of Mathematics
Department of Mathematics
Garmsar Branch, Islamic Azad University
Garmsar, Iran
E-mail: abasalt.bodaghi@gmail.com

Abbas Zivari-Kazempour
Assistant Professor of Mathematics
Department of Mathematics
Ayatollah Borujerdi University
Borujerd, Iran
E-mail: zivari@abru.ac.ir