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# A Note on Moment Generating Function of a Linear Combination of Order Statistics from a Bivariate Laplace Distribution

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**Abstract.** In this note we propose an extended skew-Laplace distribution. We obtain explicit expressions for moment generating function and the two first moments of this distribution. Next, we show that the distribution of a linear combination of order statistics from a bivariate Laplace distribution can be expressed as a mixture of extended skew-Laplace distributions. This mixture representation enables us to derive moment generating function and moments of this linear combination.

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## 1. Introduction

Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density function (PDF) and cumulative distribution function (CDF) of the standard normal distribution, respectively, then, a random variable $Z$ is said to have a standard skew-normal distribution with parameter $\lambda \in \mathbb{R}$, denoted by $Z \sim SN(\lambda)$, if its PDF is given by [see for example Azzalini (1985)]

$$\phi_{SN}(z;\lambda) = 2\phi(z)\,\Phi(\lambda z), \qquad z \in \mathbb{R}. \tag{1}$$

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There is an extension of the skew-normal distribution in (1) with an additional shape parameter [see for example Arnold and Beaver (2002)]. Specifically, a random variable $Z$ is said to have a standard extended skew-normal distribution with parameters $\lambda, \gamma \in \mathbb{R}$, denoted by $Z \sim ESN(\lambda, \gamma)$, if

$$Z \stackrel{d}{=} Z_1 | (Z_2 < \lambda Z_1 + \gamma), \quad (2)$$

where "$\stackrel{d}{=}$" denotes equality in distribution, and $Z_1$ and $Z_2$ are independent identically distributed (i.i.d) as $N(0,1)$ (standard normal distribution). The PDF, moment generating function (MGF) and mean of $Z \sim ESN(\lambda, \gamma)$ are

$$\phi_{ESN}(z; \lambda, \gamma) = \frac{\phi(z)\,\Phi(\lambda z + \gamma)}{\Phi\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)}, \quad z, \lambda, \gamma \in \mathbb{R}, \quad (3)$$

$$M_{ESN}(s; \lambda, \gamma) = \frac{\exp(\frac{s^2}{2})\Phi\left(\frac{\lambda s+\gamma}{\sqrt{1+\lambda^2}}\right)}{\Phi\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)}, \quad s \in \mathbb{R}, \quad (4)$$

and

$$E(Z) = \frac{\lambda}{\sqrt{1+\lambda^2}} \frac{\phi\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)}{\Phi\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)}, \quad (5)$$

respectively.

More generally, if the random vector $(Z_1, Z_2)^T \sim EC_2(\mathbf{0}_2, \mathbf{I}_2, h^{(2)})$ [ bivariate elliptical distribution, with location vector $\mathbf{0}_2 = (0,0)^T$,dispersion matrix $\mathbf{I}_2$ (identity matrix of dimension 2), and the density generator $h^{(2)}$], then the random variable $Z$ with stochastic representation in (2) is said to have a univariate extended skew-elliptical distribution with parameters $\lambda, \gamma \in \mathbb{R}$ and density generator $h^{(2)}$, and denoted by $Z \sim ESEC\left(\lambda, \gamma, h^{(2)}\right)$ . It is easy to show that the PDF of $Z$ in this general case is

$$f_{ESEC}\left(x; \lambda, \gamma, h^{(2)}\right) = \frac{f_{EC_1}\left(x; h^{(1)}\right) F_{EC_1}\left(\lambda x + \gamma; h^{(1)}_{x^2}\right)}{F_{EC_1}\left(\frac{\gamma}{\sqrt{1+\lambda^2}}; h^{(1)}\right)}, \quad (6)$$

where $h^{(1)}$ and $h^{(1)}_{x^2}$ can be expressed in terms of $h^{(2)}$ (for example $h^{(1)}_{x^2}(u) = \frac{h^{(2)}\left(u+x^2\right)}{h^{(1)}(x^2)}, \quad u \geqslant 0$), $f_{EC_1}\left(\cdot; h^{(1)}\right)$ is the PDF of $EC\left(0, 1, h^{(1)}\right)$, and $F_{EC_1}\left(\cdot; h^{(1)}\right)$ and $F_{EC_1}\left(\cdot; h^{(1)}_{x^2}\right)$ are the CDFs of $EC\left(0, 1, h^{(1)}\right)$ and $EC\left(0, 1, h^{(1)}_{x^2}\right)$, respectively.

Except normal distribution, two more important elliptical distributions are Student $t$ distribution and Laplace distribution. Jamalizadeh et al. (2009a) discussed the $t$ case and obtained explicit expressions for the two first moments of the extended skew-$t$ distribution. Jamalizadeh et al. (2009b) derived a recurrence relation for CDF of the extended skew-$t$ distribution and used this recurrence relation to obtain a recurrence relation for CDF of order statistics from a bivariate $t$ distribution. Our motivation for this note is to consider the Laplace case. After giving the definition of an extended skew-Laplace distribution we derive explicit expressions for MGF and the two first moments of this extended skew-Laplace distribution. Then, by using the relationship between this extended distribution and distribution of a linear combination of order statistics from a bivariate Laplace distribution, we obtain explicit expressions for MGF and the two first moments of this linear combination.

A key motivation for this note comes from vision research where a single measure of visual acuity is made in each eye, $X_1$ and $X_2$ say. A person's vision total impairment is defined as the $L$-statistic $TI = \frac{3}{4}X_{(1)} + \frac{1}{4}X_{(2)}$, where the extremes of visual acuity are $X_{(1)} = \min\left(X_1, X_2\right)$ and $X_{(2)} = \max\left(X_1, X_2\right)$, see Viana (1998) and references therein. A bivariate normal distribution is commonly assumed for $\left(X_1, X_2\right)^T$. The assumption of joint normality for the vector $\left(X_1, X_2\right)^T$ is failed in many cases. The results in this paper enables us to derive the exact distribution of $TI$ when $\left(X_1, X_2\right)^T$ follows a bivariate Laplace distribution. Distributions of order statistics from a bivariate normal random vector has been studied by Cain (1994), Cain and Pan (1995), Loperfido (2002) and Genc (2006).

In Section 2, we consider a univariate extended skew-Laplace and derive its MGF and the two first moments. In Section 3, we show that the distribution of a linear combination of order statistics from a bivariate Laplace distribution can be expressed as a mixture of the extended skew-Laplace distributions and then, by using this mixture representation, we present explicit expressions for MGF and the mean of this linear combination.

## 2. The Univariate Extended Skew–Laplace Distribution

A univariate random variable $X$ is said to have a Laplace distribution with location parameter $\mu \in \mathbb{R}$ and scale parameter $\sigma > 0$, denoted by $X \sim L(\mu, \sigma)$, if its PDF is

$$f_L(x; \mu, \sigma) = \frac{1}{2\sigma} e^{-\frac{1}{\sigma}|x-\mu|}, \qquad x \in \mathbb{R}.$$

It is easy to show that $X$ is a scale mixture of normal distribution as

$$X \stackrel{d}{=} \mu + \sqrt{W} Z,$$

where $Z \sim N\left(0, \sigma^2\right)$ is independent of $W \sim \chi_2^2$ (chi-square distribution with two degrees of freedom). If $X \sim L(0,1)$, then MGF of $X$ is

$$M_L(s) = \frac{1}{1-s^2}, \quad -1 < s < 1. \tag{7}$$

More generally, $n$-dimensional random vector $\mathbf{X} = (X_1, X_2, ..., X_n)^T$ is said to have multivariate Laplace distribution with location vector $\boldsymbol{\mu}$ and dispersion matrix $\boldsymbol{\Sigma}$, denoted by $\mathbf{X} \sim L_n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, if

$$\mathbf{X} \stackrel{d}{=} \boldsymbol{\mu} + \sqrt{W}\mathbf{Z}, \tag{8}$$

where $\mathbf{Z} \sim N_n(\mathbf{0}, \boldsymbol{\Sigma})$ independently of $W \sim \chi_2^2$.
For the stochastic representation in (8) let us to derive the PDF of $\mathbf{X}$ as

$$f_{\mathbf{L}_n}(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = (2\pi)^{-\frac{n}{2}} \mid \boldsymbol{\Sigma} \mid^{-\frac{1}{2}} (Q(\mathbf{x}))^{1-\frac{n}{2}} K_{1-\frac{n}{2}}(Q(\mathbf{x})), \tag{9}$$

where $Q(\mathbf{x}) = \sqrt{(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}-\boldsymbol{\mu})}$ and $K_v(x)$ is the Bessel functions of the third kind with index $\upsilon$, defined as

$$K_{\upsilon}(x) = \frac{1}{2}\int_0^{\infty} u^{\upsilon-1} e^{-\frac{1}{2}x\left(u^{-1}+u\right)} du, \qquad x > 0.$$

**Definition 2.1.** *(Univariate Extended Skew-Laplace Distribution). Let $(X_1, X_2)^T \sim L_2(\mathbf{0}_2, \mathbf{I}_2)$; the univariate random variable $X$ is said to have an extended skew-Laplace distribution with parameters $\lambda, \gamma \in \mathbb{R}$ , denoted by $X \sim ESL(\lambda, \gamma)$, if*

$$X \overset{d}{=} X_1 \mid (X_2 < \lambda X_1 + \gamma). \tag{10}$$

*By stochastic representation in equation (10) and the MGF of the extended skew-normal in equation (4), we can easily obtain the MGF of the extended skew-Laplace in (10). First, we need the following Lemma.*

**Lemma 2.2.** *For all $a, b \in \mathbb{R}$ $(a, b \neq 0)$, we have*

*i)* $\int_0^{\infty} e^{-\frac{1}{2}\left(a^2u^2+b^2u^{-2}\right)} du = \sqrt{\frac{\pi}{2}}\frac{e^{-|ab|}}{|a|}$,

*and*

*ii)* $\int_0^{\infty} u^{-2} e^{-\frac{1}{2}\left(a^2u^2+b^2u^{-2}\right)} du = \sqrt{\frac{\pi}{2}}\frac{e^{-|ab|}}{|b|}$.

**Proof.** See for example Lemma 1 of Datta and Ghosh (2007). □

**Theorem 2.3.** *The MGF of $X$ is, for $-1 < s < 1$,*

$$M_{ESL}(s; \lambda, \gamma) = \frac{1}{2F_L\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)(1-s^2)}$$

$$\begin{cases} \left(1 + \frac{B}{\sqrt{1+B^2}}\right) e^{A\left(\sqrt{1+B^2}-B\right)}, & \gamma \leqslant 0 \\ 2 - \left(1 - \frac{B}{\sqrt{1+B^2}}\right) e^{-A\left(\sqrt{1+B^2}+B\right)}, & \gamma > 0, \end{cases} \tag{11}$$

*where $F_L(\cdot)$ is the CDF of $L(0,1)$, and*
$A = \gamma\sqrt{\frac{1-s^2}{1+\lambda^2}}$ , $B = \frac{\lambda s}{\sqrt{(1+\lambda^2)(1-s^2)}}$.

**Proof.** From (8) and (10), we have

$$\begin{aligned} M_{ESL}(s;\lambda,\gamma) &= E\left(e^{sX_1} \mid X_2 < \lambda X_1 + \gamma\right) \\ &= \frac{1}{2}\int_0^\infty e^{-\frac{w}{2}} E\left(e^{sw^{\frac{1}{2}}Z_1} \mid Z_2 < \lambda Z_1 + \gamma w^{-\frac{1}{2}}\right) dw, \end{aligned}$$

where $Z_1$ and $Z_2$ are i.i.d $N(0,1)$. Now by using (4), we obtain

$$M_{ESL}(s;\lambda,\gamma) = \frac{1}{2F_L\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)} \int_0^\infty e^{-\frac{1}{2}(1-s^2)w} \Phi\left(\frac{\gamma w^{-\frac{1}{2}} + \lambda s w^{\frac{1}{2}}}{\sqrt{1+\lambda^2}}\right) dw.$$

Next, upon integrating by parts we have, for $-1 < s < 1$,

$$M_{ESL}(s;\lambda,\gamma) = \frac{\sqrt{2\pi}}{F_L\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)(1-s^2)}$$

$$\begin{cases} \int_0^\infty \left(B - Au^{-2}\right)\phi(u)\,\phi\left(Au^{-1} + Bu\right) du, & \gamma < 0 \\ \frac{1}{2\sqrt{2\pi}} + B\int_0^\infty \phi(u)\,\phi(Bu)\, du, & \gamma = 0 \\ \frac{1}{\sqrt{2\pi}} + \int_0^\infty \left(B - Au^{-2}\right)\phi(u)\,\phi\left(Au^{-1} + Bu\right) du, & \gamma > 0. \end{cases} .$$

Now, by using the results in Lemma 2.2 and some simple calculation we can obtain the expression in (11). □

We can readily obtain the moments of $X \sim ESL(\lambda,\gamma)$, from the derivatives of the expression of the MGF given in (11) .For example,we obtain

$$E(X) = \frac{\lambda}{\sqrt{1+\lambda^2}} \frac{f_L\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)}{F_L\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)} \tag{12}$$

and

$$E\left(X^2\right) = \frac{1}{F_L\left(\frac{\gamma}{\sqrt{1+\lambda^2}}\right)} \begin{cases} 1 - \frac{\lambda^2}{1+\lambda^2} e^{\frac{\gamma}{\sqrt{1+\lambda^2}}}, & \gamma < 0 \\ 1, & \gamma = 0 \\ 1 + \frac{3}{2}\frac{\lambda^2}{1+\lambda^2} e^{-\frac{\gamma}{\sqrt{1+\lambda^2}}} - \frac{1}{2}\frac{\lambda^2}{1+\lambda^2} e^{\frac{-2\gamma}{\sqrt{1+\lambda^2}}}, & \gamma > 0. \end{cases} \tag{13}$$

where $f_L(\cdot)$ and $F_L(\cdot)$ denote the PDF and CDF of $L(0,1)$.

## 3. Moment Generating Function of a Linear Combination of Order Statistics from a Bivariate Laplace Distribution

In this section, we show that the distribution of a linear combination of order statistics from a bivariate Laplace distribution can be expressed as a mixture of the univariate extended skew-Laplace distributions discussed in the preceding section. This mixture representation enables us to obtain the MGF and moments of this linear combination in terms of MGF and moments of the extended skew-Laplace distribution. For this purpose, assume that $(X_1, X_2)^T \sim L_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where

$$\boldsymbol{\mu} = (\mu_1, \mu_2)^T \qquad \text{and} \qquad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}, \tag{14}$$

with $\mu_1, \mu_2 \in \mathbb{R}$, $\sigma_1, \sigma_2 > 0$, and $|\rho| < 1$. Further, let $T = a_1 X_{(1)} + a_2 X_{(2)}$ $(a_1, a_2 \in \mathbb{R}, a_1 + a_2 \neq 0)$,
where $X_{(1)} = \min(X_1, X_2) < X_{(2)} = \max(X_1, X_2)$ denotes the corresponding order statistics from $(X_1, X_2)^T$,
More generally, if $(X_1, X_2)^T \sim EC_2\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}, h^{(2)}\right)$, then by using the result in Jamalizadeh et al. (2009a), we have the following Lemma.

**Lemma 3.1.** *In the elliptical case, the CDF of $T$ is the mixture, for $t \in \mathbb{R}$,*

$$\begin{aligned} & F_T\left(t; \boldsymbol{\mu}, \boldsymbol{\Sigma}, h^{(2)}\right) \\ = \; & \omega F_{ESEC}\left(\frac{t - (a_1\mu_2 + a_2\mu_1)}{\sqrt{\xi_2}}; \lambda_1, \gamma_1, h^{(2)}\right) \\ + \; & (1-\omega) F_{ESEC}\left(\frac{t - (a_1\mu_1 + a_2\mu_2)}{\sqrt{\xi_3}}; \lambda_2, \gamma_2, h^{(2)}\right), \end{aligned}$$

*where $\omega = F_{EC}\left(\frac{\mu_1 - \mu_2}{\sqrt{\xi_1}}; h^{(1)}\right)$, and $F_{ESEC}\left(\cdot; \lambda, \gamma, h^{(2)}\right)$ denotes the CDF of $Z \sim ESEC\left(\lambda, \gamma, h^{(2)}\right)$, and*

$$
\begin{aligned}
\xi_1 &= \sigma_1^2+\sigma_2^2-2\rho\sigma_1\sigma_2,\\
\xi_2 &= a_2^2\sigma_1^2+a_1^2\sigma_2^2+2a_1a_2\rho\sigma_1\sigma_2,\\
\xi_3 &= a_1^2\sigma_1^2+a_2^2\sigma_2^2+2a_1a_2\rho\sigma_1\sigma_2,\\
\lambda_1 &= \frac{\eta_1}{\sqrt{1-\eta_1^2}}, \qquad \gamma_1=\frac{\mu_1-\mu_2}{\sqrt{\xi_1}\sqrt{1-\eta_1^2}},\\
\lambda_2 &= \frac{\eta_2}{\sqrt{1-\eta_2^2}}, \qquad \gamma_2=\frac{\mu_2-\mu_1}{\sqrt{\xi_1}\sqrt{1-\eta_2^2}},
\end{aligned}
$$

*where*

$$
\begin{aligned}
\eta_1 &= \frac{\rho\sigma_1\sigma_2\left(a_1-a_2\right)-a_1\sigma_2^2+a_2\sigma_1^2}{\sqrt{\xi_1\xi_2}},\\
\eta_2 &= \frac{\rho\sigma_1\sigma_2\left(a_1-a_2\right)-a_1\sigma_1^2+a_2\sigma_2^2}{\sqrt{\xi_1\xi_3}}.
\end{aligned}
$$

**Corollary 3.2.** *In the Laplace case, the CDF of $T$ is the mixture, for $t\in\mathbb{R}$,*

$$
\begin{aligned}
& F_T\left(t;\boldsymbol{\mu},\boldsymbol{\Sigma}\right)\\
= \;& \omega F_{ESL}\left(\frac{t-\left(a_1\mu_2+a_2\mu_1\right)}{\sqrt{\xi_2}};\lambda_1,\gamma_1\right)\\
+ \;& \left(1-\omega\right)F_{ESL}\left(\frac{t-\left(a_1\mu_1+a_2\mu_2\right)}{\sqrt{\xi_3}};\lambda_2,\gamma_2\right),
\end{aligned}
\tag{15}
$$

*where $\omega=F_L\left(\frac{\mu_1-\mu_2}{\sqrt{\xi_1}}\right)$, and $F_{ESL}\left(\cdot;\lambda,\gamma\right)$ denotes the CDF of $ESL\left(\lambda,\gamma\right)$, and $\xi_1,\xi_2,\xi_3,\lambda_1,\lambda_2,\eta_1$ and $\eta_2$ are all as given in Lemma 3.1.*

**Remark 3.3.** *Upon differentiating the expression of the CDF presented in (15), we readily obtain the PDF of $T$ as, for $t\in\mathbb{R}$,*

$$
\begin{aligned}
& f_T\left(t;\boldsymbol{\mu},\boldsymbol{\Sigma}\right)\\
= \;& \frac{\omega}{\sqrt{\xi_2}}f_{ESL}\left(\frac{t-\left(a_1\mu_2+a_2\mu_1\right)}{\sqrt{\xi_2}};\lambda_1,\gamma_1\right)\\
+ \;& \frac{\left(1-\omega\right)}{\sqrt{\xi_3}}f_{ESL}\left(\frac{t-\left(a_1\mu_1+a_2\mu_2\right)}{\sqrt{\xi_3}};\lambda_2,\gamma_2\right),
\end{aligned}
\tag{16}
$$

*where* $f_{ESL}(\cdot;\lambda,\gamma)$ *denotes the PDF of* $ESL(\lambda,\gamma)$.

**Corollary 3.4.** *In the Laplace case, by using (16), the MGF of* $T$ *is, for* $-1<s<1$,

$$\begin{aligned} & M_T(s;\boldsymbol{\mu},\boldsymbol{\Sigma}) \\ = \;& \omega e^{s(a_1\mu_2+a_2\mu_1)} M_{ESL}\left(s\sqrt{\xi_2};\lambda_1,\gamma_1\right) \\ + \;& (1-\omega)\, e^{s(a_1\mu_1+a_2\mu_2)} M_{ESL}\left(s\sqrt{\xi_3};\lambda_2,\gamma_2\right), \end{aligned} \tag{17}$$

*where* $M_{ESL}(\cdot;\lambda,\gamma)$ *is given by (11).*
*We can readily obtain the moments of* $T$*, from the derivatives of the expression of the MGF given in (17). For example*

$$\begin{aligned} E(T) \;=\; & \omega\left((a_1\mu_2+a_2\mu_1)+\frac{\lambda_1\sqrt{\xi_2}}{\sqrt{1+\lambda_1^2}}\frac{f_L\left(\frac{\gamma_1}{\sqrt{1+\lambda_1^2}}\right)}{F_L\left(\frac{\gamma_1}{\sqrt{1+\lambda_1^2}}\right)}\right) \\ & +(1-\omega)\left((a_1\mu_1+a_2\mu_2)+\frac{\lambda_2\sqrt{\xi_3}}{\sqrt{1+\lambda_2^2}}\frac{f_L\left(\frac{\gamma_2}{\sqrt{1+\lambda_2^2}}\right)}{F_L\left(\frac{\gamma_2}{\sqrt{1+\lambda_2^2}}\right)}\right). \end{aligned} \tag{18}$$

*Of course by taking* $a_1=0$ $(a_2=0\ )$ *in (17) and (18) , we can easily derive MGF and the mean of* $X_{(2)}$ $X_{(1)}$*, respectively.*

**Remark 3.5.** *In the special case when*

$$(X_1,X_2)^T \sim L_2\left(\left(\begin{array}{c}\mu\\ \mu\end{array}\right),\sigma^2\left(\begin{array}{cc}1 & \rho\\ \rho & 1\end{array}\right)\right),\ \mu\in\mathbb{R},\sigma>0 \text{ and } |\rho|<1,$$

*then*

$$\frac{T-(a_1+a_2)\,\mu}{\sigma\sqrt{a_1^2+a_2^2+2\rho a_1 a_2}} \sim ESL\left(\frac{(a_2-a_1)}{|a_2+a_1|}\sqrt{\frac{1-\rho}{1+\rho}}\right).$$

## 4. Concluding Remarks

Distributions of order statistics and linear combinations of order statistics from multivariate normal and multivariate elliptical distributions have been discussed in the literature. Arellano-Valle and Genton (2007, 2008) presented the exact distributions of the largest order statistic and linear combinations of order statistics from multivariate elliptical distributions, while Jamalizadeh and Balakrishnan (2010) derived the exact distributions of order statistics and linear combinations of order statistics from elliptical distributions in terms of multivariate unified skew-elliptical distributions.They also presented explicit results for the cases when the kernel distributions are normal and $t$.

In this note by using an extended skew-Laplace distribution we study the distribution of a linear combination of order statistics from a bivariate Laplace distribution. If $\mathbf{X} = (X_1, ..., X_n)^T \sim L_n(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $X_{(1)} < X_{(2)} < .... < X_{(n)}$ denotes the corresponding order statistics from $\mathbf{X}$. It will be interesting to find explicit expressions for MGF and moments of these order statistics and linear combinations of these order statistics.

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