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Generalized T-extensions in Locally Compact Abelian Groups

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Abstract. Let $\pounds$ be the category of all locally compact abelian (LCA) groups. Let $G \in \pounds$ and $H \subseteq G$. The maximal torsion subgroup of G is denoted by tG and the closure of H by $\overline{H}$. A proper short exact sequence $0 \to A \xrightarrow{\phi} B \xrightarrow{\psi} C \to 0$ in $\pounds$ is said to be a generalized t-extension if $0 \to \overline{tA} \xrightarrow{\phi} \overline{tB} \xrightarrow{\psi} \overline{tC} \to 0$ is a proper short exact sequence. We show that the set of all generalized t-extensions of a torsion group $A \in \pounds$ by a compact group $C \in \pounds$ is a subgroup of $Ext(C, A)$. We establish conditions under which the generalized t-extensions split.

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1 Introduction

Throughout, all groups are Hausdorff topological abelian groups and will be written additively. Let $\pounds$ denote the category of locally compact abelian (LCA) groups with continuous homomorphisms as morphisms and $\Re$, the category of discrete abelian groups. A morphism is called proper if it is open onto its image, and a short exact sequence

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$0 \to A \xrightarrow{\phi} B \xrightarrow{\psi} C \to 0$ in $\pounds$ is said to be proper exact if ϕ and ψ are proper morphisms. In this case the sequence is called an extension of A by C (in $\pounds$). Following [5], we let $Ext(C, A)$ denote the group of extensions of A by C. In [1], we introduced the concept of a t-extension in $\Re$. An extension $0 \to A \xrightarrow{\phi} B \xrightarrow{\psi} C \to 0$ in $\Re$ is called a t-extension if $0 \to tA \xrightarrow{\phi|_{tA}} tB \xrightarrow{\psi|_{tB}} tC \to 0$ is an extension [1]. Let $A, C \in \Re$ and, $Ext_t(C, A)$ be the group of t-extensions of A by C [1]. In this paper, we generalize the concept of t-extension.
An extension $0 \to A \xrightarrow{\phi} B \xrightarrow{\psi} C \to 0$ in $\pounds$ will be called a generalized t-extension if $0 \to \overline{tA} \xrightarrow{\phi|_{\overline{tA}}} \overline{tB} \xrightarrow{\psi|_{\overline{tB}}} \overline{tC} \to 0$ is an extension. Let $Ext_{\bar{t}}(C, A)$ denote the set of all generalized t-extensions of A by C. Clearly, $Ext_{\bar{t}}(C, A) = Ext_t(C, A)$ for groups $A, C \in \Re$. In Section 2, we show that if A is a torsion group and C a compact group, then $Ext_{\bar{t}}(C, A)$ is a subgroup of $Ext(C, A)$ (see Theorem (2.15)). In Section 3, we establish some results on splitting of generalized t-extensions (see Lemma (3.2),(3.3),(3.4),(3.5),(3.6),(3.7)).

The additive topological group of real numbers is denoted by $\mathbb{R}$, $\mathbb{Q}$ is the group of rationals with the discrete topology, $\mathbb{Z}$ is the group of integers and, $\mathbb{Z}(n)$ is the cyclic group of order n. For any group G and H, 1_G is the identity map $G \to G$ and $Hom(G, H)$ is the group of all continuous homomorphisms from G to H, endowed with the compact-open topology. The dual group of G is $\hat{G} = Hom(G, \mathbb{R}/\mathbb{Z})$. For more on locally compact abelian groups, see [7].

2 Generalized T-extensions

Let $A, C \in \pounds$. In this section, we define the concept of a generalized t-extension of A by C. We show that the set of all generalized t-extensions of a torsion group A by a compact group C form a subgroup of $Ext(C, A)$.

Definition 2.1. *An extension $0 \to A \xrightarrow{\phi} B \xrightarrow{\psi} C \to 0$ in $\pounds$ is called a generalized t-extension if $0 \to \overline{tA} \to \overline{tB} \to \overline{tC} \to 0$ is an extension.*

Remark 2.2. Every extension of a torsion-free (or a torsion) group by a torsion free (or a torsion) group is a generalized t-extension.

Lemma 2.3. *Let A be a compact torsion group and C a compact group. Then, every extension of A by C is a generalized t-extension.*

Proof. Let $0 \to A \xrightarrow{\phi} B \xrightarrow{\psi} C \to 0$ be an extension of A by C. First, we show that $\psi(tB) = tC$. Let $c \in tC$. Then, there exists a positive integer n such that $nc = 0$. Since $\psi : B \to C$ is surjective, so $\psi(b) = c$ for some $b \in B$. Hence, $nb \in ker\psi = im\phi$. So, $\phi(a) = nb$ for some $a \in A$. Since A is a torsion group, so $ma = 0$ for some positive integer m. Hence, $(mn)b = 0$ and $b \in tB$. Since $\phi(A)$ is a compact subgroup of B and $B/\phi(A) \cong C$, so ψ is closed (See Theorem 5.18 of [7]). Hence, $\psi(\overline{tB}) = \overline{\psi(tB)} = \overline{tC}$. Since A is torsion, so $Ker\psi \mid_{\overline{tB}} \subseteq Im\phi$. Hence, $0 \to A \xrightarrow{\phi} \overline{tB} \xrightarrow{\psi} \overline{tC} \to 0$ is an exact sequence. By Theorem 5.29 of [7], $\phi : A \to \overline{tB}$ and $\psi : \overline{tB} \to \overline{tC}$ are proper morphisms. Hence, $0 \to A \xrightarrow{\phi} \overline{tB} \xrightarrow{\psi} \overline{tC} \to 0$ is an extension.
□

The dual of an extension $E : 0 \to A \to B \to C \to 0$ is defined by $\hat{E} : 0 \to \hat{C} \to \hat{B} \to \hat{A} \to 0$. The following example shows that the dual of a generalized t-extension need not be a generalized t-extension.

Example 2.4. Consider the extension $E : 0 \to \mathbb{Z}_2 \to \mathbb{R}/\mathbb{Z} \xrightarrow{\times 2} \mathbb{R}/\mathbb{Z} \to 0$. By Lemma 2.3, E is a generalized t-extension. But, $\hat{E} : 0 \to \mathbb{Z} \xrightarrow{\times 2} \mathbb{Z} \to \mathbb{Z}_2 \to 0$ is not a generalized t-extension.

Recall that two extensions $0 \to A \xrightarrow{\phi_1} B \xrightarrow{\psi_1} C \to 0$ and $0 \to A \xrightarrow{\phi_2} X \xrightarrow{\psi_2} C \to 0$ are said to be equivalent if there is a topological isomorphism $\beta : B \to X$ such that the following diagram

$$\begin{tikzcd} 0 \arrow[r] & A \arrow[r, "\phi_1"] \arrow[d, "1_A"] & B \arrow[r, "\psi_1"] \arrow[d, "\beta"] & C \arrow[r] \arrow[d, "1_C"] & 0 \\ 0 \arrow[r] & A \arrow[r, "\phi_2"] & X \arrow[r, "\psi_2"] & C \arrow[r] & 0 \end{tikzcd}$$

is commutative.

Lemma 2.5. *An extension equivalent to a generalized t-extension is a generalized t-extension.*

Proof. Let
$$E_1 : 0 \to A \overset{\phi_1}{\to} B \overset{\psi_1}{\to} C \to 0$$
and
$$E_2 : 0 \to A \overset{\phi_2}{\to} X \overset{\psi_2}{\to} C \to 0$$
be two equivalent extensions such that E_1 is a generalized t-extension. Then, there is a topological isomorphism $\beta : B \to X$ such that $\beta\phi_1 = \phi_2$ and $\psi_2\beta = \psi_1$. Since $\beta(\overline{tB}) = \overline{tX}$, so
$$\psi_2(\overline{tX}) = \psi_2\beta(\overline{tB}) = \psi_1(\overline{tB}).$$
On the other hand, E_1 is a generalized t-extension. Hence, $\psi_2(\overline{tX}) = \overline{tC}$ and $\psi_2 \mid_{\overline{tX}}$: $\overline{tX} \to \overline{tC}$ is surjective. Since $\phi_2 = \beta\phi_1$ and E_1 is a generalized t-extension, so
$$\psi_2\phi_2(\overline{tA}) = \psi_2(\beta\phi_1(\overline{tA})) = \psi_1\phi_1(\overline{tA}) = 0.$$
Hence, $Im\phi_2 \mid_{\overline{tA}} \subseteq Ker\psi_2 \mid_{\overline{tX}}$. Now, we show that $Ker\psi_2 \mid_{\overline{tX}} \subseteq Im\phi_2 \mid_{\overline{tA}}$. Let $x \in \overline{tX}$ and $\psi_2(x) = 0$. Then, there exists $b \in \overline{tB}$ such that $x = \beta(b)$. Since $\psi_1(b) = \psi_2\beta(b) = \psi_2(x) = 0$ and E_1 is a generalized t-extension, so $b = \phi_1(a)$ for some $a \in \overline{tA}$. Hence
$$\phi_2(a) = \beta\phi_1(a) = \beta(b) = x$$
and $0 \to \overline{tA} \overset{\phi_2}{\to} \overline{tX} \overset{\psi_2}{\to} \overline{tC} \to 0$ is an exact sequence. Since
$$\psi_2 \mid_{\overline{tX}} = \psi_1 \mid_{\overline{tB}} (\beta \mid_{\overline{tB}})^{-1}, \phi_2 \mid_{\overline{tA}} = \beta \mid_{\overline{tB}} (\phi_1 \mid_{\overline{tA}})$$
$\psi_1 \mid_{\overline{tB}}$ and $\phi_1 \mid_{\overline{tA}}$ are open, so E_2 is a generalized t-extension. $\square$

Lemma 2.6. *Let $C \in £$ be a compact group, $0 \to A \overset{\phi}{\to} B \overset{\psi}{\to} C \to 0$ be a generalized t-extension and assume*

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A & \xrightarrow{\mu} & X & \xrightarrow{\nu} & Y & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle 1_A} & & \downarrow{\scriptstyle \theta} & & \downarrow{\scriptstyle f} & & \\ 0 & \longrightarrow & A & \xrightarrow{\phi} & B & \xrightarrow{\psi} & C & \longrightarrow & 0 \end{array}$$

is the standard pullback diagram in £ (See [5]). If Y be a $\sigma-$ compact group in £, then
$$0 \to A \overset{\mu}{\to} X \overset{\nu}{\to} Y \to 0$$
is a generalized t-extension.

Proof. We have

$$X = \{(y,b) \in Y \bigoplus B : f(y) = \psi(b)\}.$$

and

$$\mu : a \mapsto (0, \phi(a)), \nu : (y,b) \mapsto y, \theta : (y,b) \mapsto b.$$

Since Y is a $\sigma-$ compact group, so $\overline{tY}$ is $\sigma-$ compact. By Theorem 5.29 of [7], $f : \overline{tY} \to \overline{tC}$ is a proper morphism. Now, consider the following standard pullback diagram

$$\begin{tikzcd}
0 \arrow[r] & \overline{tA} \arrow[r, "\phi'"] \arrow[d] & N \arrow[r, "\psi'"] \arrow[d] & \overline{tY} \arrow[r] \arrow[d, "f"] & 0 \\
0 \arrow[r] & \overline{tA} \arrow[r, "\phi"] & \overline{tB} \arrow[r, "\psi"] & \overline{tC} \arrow[r] & 0
\end{tikzcd}$$

where $N = \{(y,b) \in \overline{tY} \bigoplus \overline{tB} : f(y) = \psi(b)\}$. First, we show that $N = \overline{tX}$. Clearly $N \subseteq \overline{tX}$. Suppose that $(y,b) \in \overline{tX}$. We shall show that $y \in \overline{tY}$ and $b \in \overline{tB}$. Let W be a neighborhood of y in Y and V a neighborhood of b in B. Then, $(W \times V) \bigcap X$ is a neighborhood of (y,b) in X. So, $(W \times V) \bigcap tX \neq \emptyset$. Assume that $(y_1, b_1) \in (W \times V) \bigcap tX$. Then,$y_1 \in W \bigcap tY$ and $b_1 \in V \bigcap tB$. Clearly, $\phi' = \mu \mid_{\overline{tA}}$ and $\psi' = \nu \mid_{\overline{tX}}$. Hence, $0 \to \overline{tA} \overset{\mu}{\to} \overline{tX} \overset{\nu}{\to} \overline{tY} \to 0$ is an extension. $\square$

Lemma 2.7. *Let $G \in \pounds$ and H be a closed, torsion subgroup of G. Then $\overline{t(G/H)} = \overline{tG}/H$.*

Proof. Let $\pi : G \to G/H$ be the natural mapping. Then, $\pi(\overline{tG}) \subseteq \overline{\pi(tG)} \subseteq \overline{t(G/H)}$. But, $\pi(\overline{tG}) = (\overline{tG} + H)/H$ and $H \subseteq \overline{tG}$. Hence, $\overline{tG}/H \subseteq \overline{t(G/H)}$. Now, suppose that $x + H \in \overline{t(G/H)}$. We show that $x \in \overline{tG}$. Let V be a neighborhood of G containing x. Then, $y + H \in (V + H)/H \bigcap t(G/H) \neq \phi$ for some $y \in G$. From $y + H \in (V + H)/H$, deduce that $y + H = z + H$ for some $z \in V$. Since H is torsion, so $ny = nz$ for some positive integer n. On the other hand, $y + H \in t(G/H)$. So, $ky \in H$ for some positive integer k. Hence, $mky = 0$ for some positive integer m. Therefore, $mknz = 0$. This shows that $z \in V \bigcap tG$. Hence, $x \in \overline{tG}$. $\square$

Corollary 2.8. *Every extension of a torsion group by a torsion-free group is a generalized t-extension.*

Proof. It is clear by Lemma 2.7. $\square$

Definition 2.9. *A group $G \in £$ is called torsion-dense if tG is dense in G.*

Lemma 2.10. *Let H be a closed, torsion subgroup of $G \in £$ such that G/H is torsion-dense. Then, G is torsion-dense.*

Proof. By Lemma 2.7, $\overline{t(G/H)} = \overline{tG}/H$. Since G/H is torsion-dense, so $\overline{tG}/H = G/H$. Hence, $\overline{tG} = G$. $\square$

Corollary 2.11. *Every extension of a torsion group by a torsion-dense group is a generalized t-extension.*

Proof. Let $E : 0 \to A \xrightarrow{\phi} B \xrightarrow{\psi} C \to 0$ be an extension in $£$ such that A and C are torsion and torsion-dense groups, respectively. Then, $\phi(A)$ is a closed, torsion subgroup of B and $B/\phi(A) \cong C$ is a torsion-dense group. So, by Lemma 2.10, B is a torsion-dense group. Hence, E is a generalized t-extension. $\square$

Lemma 2.12. *Let $A \in £$ be a torsion group. Then, a pushout of a generalized t-extension of A by C in $£$ is a generalized t-extension.*

Proof. Suppose that $E : 0 \to A \xrightarrow{\phi} B \to C \to 0$ is a generalized t-extension and $f : A \to G$ a proper morphism in $£$. Then, $fE : 0 \to G \to X \to C \to 0$ is a pushout of E, where $X = (G \bigoplus B)/H$ and $H = \{(-f(a), \phi(a)); a \in A\}$ (See [5]). Since E is a generalized t-extension, so $E' : 0 \to A \to \overline{tB} \to \overline{tC} \to 0$ is an extension. Hence, hE' is an extension where $h : A \to \overline{tG}$ defined by $h(a) = f(a)$ for every $a \in A$. But, $hE' : 0 \to \overline{tG} \to Y \to \overline{tC} \to 0$ where $Y = (\overline{tG} \bigoplus \overline{tB})/K$ and $K = \{(-h(a), \phi(a)); a \in A\}$. Clearly, $K = H$. Since H is a closed, torsion subgroup of $G \bigoplus B$, so by Lemma 2.7, $\overline{tX} = (\overline{tG} \bigoplus \overline{tB})/H = Y$. Hence, fE is a generalized t-extension. $\square$

Corollary 2.13. *Let $A \in £$ be a torsion group and $E : 0 \to A \to B \to C \to 0$ a generalized t-extension in $£$. Then, $-1_A E$ is a generalized t-extension.*

Proof. $-1_A E$ is a pushout of E. So by Lemma 2.12, it is a generalized t-extension. $\square$

Remark 2.14. Let C and A be two groups, and $0 \to A \overset{\phi_1}{\to} B_1 \overset{\psi_1}{\to} C \to 0$ and $0 \to A \overset{\phi_2}{\to} B_2 \overset{\psi_2}{\to} C \to 0$ be two generalized t-extensions of A by C. An easy calculation shows that $0 \to A \bigoplus A \overset{(\phi_1 \bigoplus \phi_2)}{\to} B_1 \bigoplus B_2 \overset{(\psi_1 \bigoplus \psi_2)}{\to} C \bigoplus C \to 0$ is a generalized t-extension where $(\phi_1 \bigoplus \phi_2)(a_1, a_2) = (\phi_1(a_1), \phi_2(a_2))$ and $(\psi_1 \bigoplus \psi_2)(b_1, b_2) = (\psi_1(b_1), \psi_2(b_2))$.

Theorem 2.15. *Let $A \in \pounds$ be a torsion group and $C \in \pounds$ a compact group. Then, the class $Ext_{\bar{t}}(C, A)$ of all equivalence classes of generalized t-extensions of A by C is an subgroup of $Ext(C, A)$ with respect to the operation defined by*

$$[E_1] + [E_2] = [\nabla_A (E_1 \bigoplus E_2) \triangle_C]$$

where E_1 and E_2 are generalized t-extensions of A by C and ∇_A and $\triangle_C$ are the diagonal and codiagonal homomorphisms.

Proof. Clearly, the trivial extension of A by C is a generalized t-extension. By Remark 2.14, Lemma 2.6 and Lemma 2.12, $[E_1] + [E_2] \in Ext_{\bar{t}}(C, A)$ for two generalized t-extensions E_1 and E_2 of A by C. So, $Ext_{\bar{t}}(C, A)$ is a subgroup of $Ext(C, A)$. $\square$

Remark 2.16. Let A and C be two discrete groups. Then, $Ext_{\bar{t}}(C, A) = Ext_t(C, A)$.

3 Splitting of Generalized T-extensions

In this section, we establish some conditions on A and C such that $Ext_{\bar{t}}(C, A) = 0$.

Definition 3.1. *A group $G \in \pounds$ is called an $\pounds-$cotorsion group if $Ex(X, G) = 0$ for every torsion-free group $X \in \pounds$ (See [4]).*

Lemma 3.2. *Let A be a discrete group. Then, $Ext_{\bar{t}}(X, A) = 0$ for every $X \in \pounds$ if and only if $A = 0$.*

Proof. Let $Ext_{\bar{t}}(X, A) = 0$ for every $X \in \pounds$. So, $Ext_{\bar{t}}(\mathbb{R}/\mathbb{Z}, A) = 0$. Consider the exact sequence $0 \to tA \overset{i}{\to} A \to A/tA \to 0$. By Corollary 2.10 of [6], we have the following exact sequence

$$Hom(\mathbb{R}/\mathbb{Z}, tA) \to Ext(\mathbb{R}/\mathbb{Z}, tA) \overset{i_*}{\to} Ext(\mathbb{R}/\mathbb{Z}, A)$$

$\mathbb{R}/\mathbb{Z}$ is a connected group and A/tA a discrete group. So, $Hom(\mathbb{R}/\mathbb{Z}, A/tA) = 0$. Hence, i_* is injective. By Lemma 2.12, $i_*(Ext_{\bar{t}}(\mathbb{R}/\mathbb{Z}, tA) \subseteq Ext_{\bar{t}}(\mathbb{R}/\mathbb{Z}, A) = 0$. So, $Ext_{\bar{t}}(\mathbb{R}/\mathbb{Z}, tA) = 0$. By Corollary 2.11, $tA \cong Ext(\mathbb{R}/\mathbb{Z}, tA) = 0$. So, A is a torsion-free group. By Remark 2.2, $Ext(X, A) = 0$ for every torsion-free group $X \in \pounds$. Hence, A is an $\pounds-$cotorsion group. By Corollary 10 of [4], $A = 0$. $\square$

Lemma 3.3. *Let $G \in \pounds$ be a torsion group. Then, $Ext_{\bar{t}}(X, G) = 0$ for every $X \in \pounds$ if and only if $G = 0$.*

Proof. Let $Ext_{\bar{t}}(X, G) = 0$ for every $X \in \pounds$. Then, $Ext_{\bar{t}}(X, G) = 0$ for every torsion-free group $X \in \pounds$. By Corollary 2.8, $Ex(X, G) = Ext_{\bar{t}}(X, G) = 0$ for every torsion-free group $X \in \pounds$. So, G is an $\pounds-$cotorsion group. By Theorem 24.30 of [7], G contains a compact open subgroup K. Consider the exact sequence $0 \to K \to G \to G/K \to 0$. By Corollary 2.10 of [6], we have the following exact sequence

$$\to Ext(\mathbb{R}/\mathbb{Z}, K) \to Ext(\mathbb{R}/\mathbb{Z}, G) \to Ext(\mathbb{R}/\mathbb{Z}, G/K) \to 0$$

By Corollary 2.11, $Ext(\mathbb{R}/\mathbb{Z}, G) = Ext_{\bar{t}}(\mathbb{R}/\mathbb{Z}, G) = 0$. Hence, $G/K \cong Ext(\mathbb{R}/\mathbb{Z}, G/K) = 0$. So, G is a compact torsion group. By Corollary 9 of [4], $G = 0$. $\square$

Lemma 3.4. *Let A be a discrete group and $Ext_{\bar{t}}(A, X) = 0$ for every $X \in \pounds$. Then, A is a direct sum of cyclic groups.*

Proof. Let A be a discrete group and $Ext_{\bar{t}}(A, X) = 0$ for every $X \in \pounds$. Then, $Ext_{\bar{t}}(A, X) = 0$ for every discrete group X. Hence, by Remark 2.16, $Ext_t(A, X) = 0$ for every group $X \in \Re$. By Theorem 3.13 of [1], A is a direct sum of cyclic groups. $\square$

Lemma 3.5. *Let G be a compact group. Then, $Ext_{\bar{t}}(G, X) = 0$ for every $X \in \pounds$ if and only if $G = 0$.*

Proof. Let G be a compact group and $Ext_{\bar{t}}(G, X) = 0$ for every $X \in \pounds$. Then, $Ext_{\bar{t}}(G, \mathbb{Z}_n) = 0$ for every positive integer n. By Lemma 2.3, $Ext(G, \mathbb{Z}_n) = 0$. So, $Ext(\mathbb{Z}_n, \hat{G}) = 0$. Hence,$\hat{G}$ is a divisible group. So, G is a torsion-free group. By Remark 2.2, $Ext(G, X) = Ext_{\bar{t}}(G, X) = 0$ for every torsion-free group $X \in \pounds$. Hence, $Ext(Y, \hat{G}) = 0$ for every divisible group $Y \in \pounds$. By Proposition 3.1 of [13], $G = 0$. $\square$

Lemma 3.6. *Let G be a torsion group. Then, $Ext_{\bar{t}}(G, X) = 0$ for every $X \in \pounds$ if and only if $G = 0$.*

Proof. Let G be a torsion group and $Ext_{\bar{t}}(G, X) = 0$ for every $X \in \pounds$. By Remark 2.2, $Ext(G, X) = 0$ for every torsion group X. Hence, by Theorem 4 of [4], $G = 0$. $\square$

Lemma 3.7. *Let $G \in \pounds$ be a torsion-free group. Then, $Ext_{\bar{t}}(G, X) = 0$ for every $X \in \pounds$ if and only if $G \cong \mathbb{R}^n \bigoplus(\bigoplus_\sigma \mathbb{Z})$, where n is a positive integer and σ a cardinal number.*

Proof. Let $G \in \pounds$ be a torsion-free group and $Ext_{\bar{t}}(G, X) = 0$ for every $X \in \pounds$. By Remark 2.2, $Ext(G, X) = 0$ for every torsion-free group X. Hence, by Theorem 2 of [4], $G \cong \mathbb{R}^n \bigoplus(\bigoplus_\sigma \mathbb{Z})$. Conversely is clear by Theorem 3.3 of [12]. $\square$

References

[1] A. Alijani and H. Sahleh, On t-extensions of abelian groups, *Khayyam J. Math.*, 5 (2019), 60-68.

[2] L. Fuchs, *Infinite Abelian Groups*, Vol. 1, Academic Press, (1970).

[3] R. O. Fulp, Homological study of purity in locally compact groups, *Proc. London Math. Soc*, 21 (1970), 501-512.

[4] R. O. Fulp, Splitting locally compact abelian groups, *Michigan Math. J*, 19 (1972), 47-55.

[5] R.O. Fulp and P. Griffith, Extensions of locally compact abelian groups I, *Trans. Amer. Math. Soc*, 154 (1971),341-356.

[6] R. O. Fulp and P. Griffith, Extensions of locally compact abelian groups II, *Trans. Amer. Math. Soc*, 154 (1971), 357-363.

[7] E. Hewitt and K. Ross, *Abstract Harmonic Analysis*, vol. 1, 2nd edn, Springer-Verlog, Berlin (1979).

[8] M. A. Khan, Chain conditions on subgroups of LCA groups, *Pacific J. Math*, 86 (1980), 517-534.

[9] J. A. Khan, The finite torsion subgroup of an LCA group need not split, *Period. Math. Hungar*, 31 (1995), 43-44.

[10] P. Loth, Splitting of the identity component in locally compact abelian groups, *Rend. Sem. Mat. Univ. Padova*, 88 (1992), 139-143.

[11] P. Loth, Pure extensions of locally compact abelian groups, *Rend. Sem. Mat. Univ. Padova*, 116 (2006), 31-40.

[12] M. Moskowitz, Homological algebra in locally compact abelian groups, *Trans. Amer. Math. Soc*, 127 (1967), 361-404.

[13] H. Sahleh and A. A. Alijani, Splitting of extensions in the category of locally compact abelian groups, *Int. J. Group Theory*, 3 (2014), 39-45.

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