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An Approach to Generate New Soft Sets and Spaces in Soft Ditopological Spaces

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Abstract. This paper is about, the detailed analysis of soft ditopological space theory (SDT - space theory) is ameliorated by introducing new soft sets called $\tilde{\delta}$-b-unclosed sets, $\tilde{\delta}$-b-closed sets and $\tilde{\delta}$-b-dense sets, which are needed for the definition of extremally disconnected spaces and submaximal spaces in soft ditopological spaces. Moreover, $\tilde{\delta}$-regular- unclosed, $\tilde{\delta}$-preunclosed, $\tilde{\delta}$-semi unclosed, $\tilde{\delta}$- α-unclosed and $\tilde{\delta}$-β- unclosed sets in SDT-space are determined and studied relations between these sets in detail. A new idea is introduced in order to prove relations, which gives an affirmative answer to understand the structure of SDT-spaces. It is demonstrated that separately these frameworks can in any case be very confounded, a perhaps increasingly tractable errand is to portray their conceivable joint dispersions by using recently characterized $\tilde{\delta}$-sets.

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1 Introduction

Vulnerability happening in fields, for example, financial aspects, building and condition, can't be taken care of utilizing conventional numerical

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apparatuses yet might be managed utilizing a wide scope of existing hypotheses, for example, likelihood hypothesis, the speculations of fluffy sets, unclear sets, interim arithmetic and harsh sets [5]. Be that as it may, as is called attention to in [3], none of these speculations can deal with all issues. Molodtsov [3] recommended that one reason might be the deficiency of the parametrization apparatus of the pertinent hypothesis and as an answer. Consequently, Molodtsov [3] presented the idea of delicate sets that is a scientific apparatus for managing vulnerabilities, that is free from the challenges that have vexed the customary hypothetical approaches. After this invention, in 2002 and 2003, very readable account of this theory has been given by Maji et al. [1, 2] on some mathematical aspects of soft sets. Over the last fifteen years or so, there have been many examples of explicit descriptions of soft sets and soft set operations flows: see e.g. [1], [2], [5], [7], [8], [9] and [10]. By using this operations, the theory of soft topological space determined by Shabir and Naz [11] over the initial universe. There are various outcomes in the writing identifying with delicate topology and one of the finest work is Aygünoglu and Aygün [12] where they introduce soft product topology and determined soft compactness. As a continuation of this, it is ingenuous to investigate the behavior of soft topological structures in soft set theoretic forms. This achieves the characteristic inquiry of whether there is any extra topology on the delicate set every single imaginable agenda. Consequently, soft bitopological and soft ditopological (SDT) space on a soft set was introduced in the works of [15], [16] and [17], respectively. The idea of SDT-space on a delicate set comprises of with two structures on it - a delicate topology and a delicate subspace topology. The first one is used to describe soft unclosedness properties of a soft topological space while the second one deals with its sub-soft unclosedness properties. Over the span of composing this paper I discovered that few creators in [13] and [14] has simultaneously obtained results on soft sets in soft bitopological spaces similar to soft set structures determined in SDT-space in certain respects.
In perspective on this and thinking about the significance of topological structure in growing delicate set hypothesis, I have introduced in this paper new soft sets called $\overline{\delta}$-b-unclosed sets, $\overline{\delta}$-b-closed sets and $\overline{\delta}$-b-dense sets, which are needed for the definition of extremally disconnected

spaces and submaximal spaces in soft ditopological spaces. In order to make a more comprehensive research, in this paper, $\overline{\delta}$-regular- unclosed, $\overline{\delta}$-preunclosed, $\overline{\delta}$-semi unclosed, $\overline{\delta}$-α-unclosed and $\overline{\delta}$-β-unclosed sets in soft ditopological space are determined and studied relations between these sets.

The point of this article is to think about the structure of these new characterized delicate sets and the connection between them for building a topological hyperspace with delicate sets. The handiness and enthusiasm of this correspondence of new characterized - delicate sets will, obviously, be upgraded if there is a method for coming back from the changes to one another, in other words, if there is a recipe that describes $\overline{\delta}$-soft sets. In the last segment, every one of the investigations work out as expected and I take up an outcome which assumes a significant job in the portrayal of this new $\overline{\delta}$-soft sets and demonstrates it by a chart.

2 Preliminary

Throughout that follows, I shall accept and permanently use elemantary definitions and preliminary results of the works Molodtsov [3], Maji et al. [1, 2], Aktas and Cagman [9], Shabir and Naz [11], Şenel [17] are displayed in this segment in this paper. Except if generally expressed, all through this paper, U alludes to an underlying universe, E is a lot of parameters and $P(U)$ is the power set of U.

Definition 2.1. *A soft set f on the universe U is a set defined by $f_A : E \to P(U)$ such that $f(e) = \emptyset$ if $e \in E\backslash A$.*
Here f_A is also called an approximate function and if $A = E$, then we use f instead of f_E.
Here f is likewise called a rough capacity. A delicate set over U can be spoken to by the arrangement of requested sets

$$f = \{(e, f(e)) : e \in E\} \tag{1}$$

I will classify any soft set f with the function $f(e)$ and I shall use that concept as interchangeable. Soft sets are indicated by the letters $f, g, h, ...$ and the corresponding functions by $f(e), g(e), h(e), ...$
Throughout this paper, the set of all soft sets over U will be indicated by

$\mathbb{S}$. From now on, all undetermined concepts about soft sets, I refer to: [19].

Definition 2.2. *Let $f \in S$. Then,*
If $f(e) = \emptyset$ for all $e \in E$, then f is named an empty set, indicated by Φ.
If $f(e) = U$ for all $e \in E$, then f is named universal simple set, indicated by $\tilde{E}$.

Definition 2.3. *Let $f, g \in S$. Then,*
If f is a simple subset of g, indicated by $f \tilde{\subseteq} g$, if $f \subseteq g$ for all $e \in E$.
If f and g are simple equal, indicated by $f = g$, if and only if $f(e) = g(e)$ for all $e \in E$.

Definition 2.4. *Let $f, g \in S$. Then, the intersection of f and g, indicated $f \tilde{\cap} g$, is determined by and the union of f and g, indicated $f \tilde{\cup} g$, is determined by*

$$(f \tilde{\cup} g)(e) = f(e) \cup g(e) \tag{2}$$

for all $e \in E$.

Definition 2.5. *Let $f \in S$. Then, the simple complement of f, indicated f^c, is determined by*

$$f^c(e) = U \tag{3}$$

$f(e)$, for all $e \in E$.

Definition 2.6. *Let $f \in S$. The power simple set of f is determined by*

$$P(f) = \{f_i \tilde{\subseteq} f : i \in I\} \tag{4}$$

and its cardinality is determined by

$$|P(f)| = 2^{\sum\limits_{e \in E} f(e)} \tag{5}$$

where $|f(e)|$ is the cardinality of $f(e)$.

Example 2.7 (19)**.** . Let $U = \{u_1, u_2, u_3\}$ and $e = \{e_1, e_2\}$, $f \in S$ and $f = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$.
Then,

$$f_1 = \{(e_1, \{u_1\})\}$$

$$
\begin{aligned}
f_2 &= \{(e_1, \{u_2\})\} \\
f_3 &= \{(e_1, \{u_1, u_2\})\} \\
f_4 &= \{(e_2, \{u_2\})\} \\
f_5 &= \{(e_2, \{u_3\})\} \\
f_6 &= \{(e_2, \{u_2, u_3\})\} \\
f_7 &= \{(e_1, \{u_1\}), (e_2, \{u_2\})\} \\
f_8 &= \{(e_1, \{u_1\}), (e_2, \{u_3\})\} \\
f_9 &= \{(e_1, \{u_1\}), (e_2, \{u_2, u_3\})\} \\
f_{10} &= \{(e_1, \{u_2\}), (e_2, \{u_2\})\} \\
f_{11} &= \{(e_1, \{u_2\}), (e_2, \{u_3\})\} \\
f_{12} &= \{(e_1, \{u_2\}), (e_2, \{u_2, u_3\})\} \\
f_{13} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_2\})\} \\
f_{14} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_3\})\} \\
f_{15} &= f \\
f_{16} &= \Phi
\end{aligned} \tag{6}
$$

are all soft subsets of f. So $|\tilde{P}(f)| = 2^4 = 16$.

Definition 2.8. *Let $f \in S$. A soft topology on f, indicated by $\tilde{\tau}$, is a collection of soft subsets of f having following properties:*
i. $f, \Phi \in \tilde{\tau}$
ii. $\{g_i\}_{i \in I} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcup}_{i \in I} g_i \in \tilde{\tau}$
iii. $\{g_i\}_{i=I}^{n} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcap}_{i=1}^{n} g_i \in \tilde{\tau}$
The pair $(f, \tilde{\tau})$ is called a soft topological space.

Example 2.9. Refer example 2.7., $\tilde{\tau}^1 = \tilde{\mathcal{P}}(f)$, $\tilde{\tau}^0 = \{\Phi, f\}$ and $\tilde{\tau} = \{\Phi, f, f_2, f_{11}, f_{13}\}$ are soft topologies on f.

Definition 2.10. *Let $(f, \tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called soft unclosed set. Clearly, Φ and f are soft unclosed sets.*

Essential thoughts and ideas of straightforward ditopological spaces, for example, basic unclosed and shut sets, basic inside, basic conclusion, basic premise, basic compliment and set up a few properties of these basic ideas are controlled by a few creators. As opposed to examine these works in full all inclusive statement, let us take a gander at a specific circumstance of this sort: In crafted by Dizman [20], the idea of basic ditopological space is presented with two structures which one identified with the property of unclosedness in the space and the other one transferred on the property of closeness in the space. This is an unmistakable logical inconsistency of the way that on the off chance that we know the straightforward topology on a basic set, we can undoubtedly get basic unclosed and basic shut sets by supplement activity. In that manner, in a straightforward ditopological space that incorporates same trademark properties created with unclosedness and closeness of one another.
In order to make a more comprehensive research, I have decided to develop the theory; considering that rather than studying with the triple structure that is built by the spaces made through one another, it is more benefical to study a triple structure that contains a different and a new space.
The thought of straightforward ditopology by Senel [17] is broader than that by Dizman [20]. The idea of basic ditopological (SDT) space on a straightforward set in [17] with two structures on it is being presented - a basic topology and a basic subspace topology. The first is utilized to depict basic unclosedness properties of a basic topological space while the subsequent one arrangements with its sub - straightforward unclosedness properties. It very well may be resolved as pursues:

Definition 2.11. *Let f be a nonempty soft set over the universe U, $g\tilde{\subseteq}f$, $\tilde{\tau}$ be a soft topology on f and $\tilde{\tau}_g$ be a soft subspace topology on g. Then, $(f, \tilde{\tau}, \tilde{\tau}_g)$ is called a soft ditopological space which is abbreviated as SDT-space.*
A pair $\tilde{\delta} = (\tilde{\tau}, \tilde{\tau}_g)$ is called a soft ditopology over f and the members of $\tilde{\delta}$ are said to be $\tilde{\delta}$-soft unclosed in f.
The complement of $\tilde{\delta}$-soft unclosed set is called $\tilde{\delta}$-soft closed soft set.

Example 2.12 (17)**.** Let us consider all soft subsets on f in the Example 2.7.
Let $\tilde{\tau} = \{\Phi, f, f_2, f_{11}, f_{13}\}$ be a soft topology on f. If $g = f_9$, then

$\tilde{\tau}_g = \{\Phi, f_5, f_7, f_9\}$ and $(g, \tilde{\tau}_g)$ is a soft topological subspace of $(f, \tilde{\tau})$. Hence, we get soft ditopology over f as $\tilde{\delta} = \{\Phi, f, f_2, f_5, f_7, f_9, f_{11}, f_{13}\}$.

Definition 2.13. *Let $h \tilde{\subseteq} f$. Then, $\tilde{\delta}$-soft interior ($\tilde{\delta}$-int) of h, indicated by $(h)^o_{\tilde{\delta}}$, is determined by*

$$(h)^o_{\tilde{\delta}} = \tilde{\bigcup}\{h : k \tilde{\subset} h, k \text{ is } \tilde{\delta}\text{-soft open}\} \tag{7}$$

The $\tilde{\delta}$-soft closure ($\tilde{\delta}$-cl) of h, indicated by ($\overline{h}_{\tilde{\delta}}$) is determined by

$$(\overline{h})_{\tilde{\delta}} = \tilde{\bigcap}\{k : h \tilde{\subset} k, k \text{ is } \tilde{\delta}\text{-soft closed}\} \tag{8}$$

Note that $(h)^o_{\tilde{\delta}}$ is the biggest $\tilde{\delta}$- -soft unclosed set that contained in h and $(\overline{h})_{\tilde{\delta}}$ is the smallest $\tilde{\delta}$-soft closed set that containing h.

3 An Approach to Generate New $\tilde{\delta}$-Soft Sets in SDT-Spaces

In this section I aim to discuss how soft set theory can be used for developing SDT-spaces by using $\tilde{\delta}$-b-unclosed set, $\tilde{\delta}$-b-closed set, $\tilde{\delta}$-b-dense set, $\tilde{\delta}$-regular- unclosed set, $\tilde{\delta}$-preunclosed set, $\tilde{\delta}$-semi unclosed set, $\tilde{\delta}$-α-unclosed set and $\tilde{\delta}$-β- unclosed set which are first mentioned in this work. In accordance with this purpose, I establish several interesting properties of these new determined $\tilde{\delta}$-soft sets also, their relationship which are central for research on basic ditopology and will fortify the establishments of the hypothesis of straightforward ditopological spaces.

Definition 3.1. *Let $(f, \tilde{\delta})$ be a SDT-space, $h \tilde{\subseteq} f$. Then h is called $\tilde{\delta}$-simple-b-unclosed set (briefly $\tilde{\delta}$-sb-unclosed) if $h \tilde{\subseteq} \tilde{\delta}$-int $(\tilde{\delta} - cl(h)) \tilde{\cap} \tilde{\delta}$-cl $(\tilde{\delta} - int(h))$. The set of all $\tilde{\delta}$-simple-b-unclosed sets are indicated by $\tilde{\delta}$-$SbO(f)$.*
The complement of $\tilde{\delta}$- soft b-unclosed set is called $\tilde{\delta}$-soft b-closed set.

Definition 3.2. *A soft subset h of a SDT-space $(f, \tilde{\delta})$ is called $\tilde{\delta}$-soft dense if $\tilde{\delta}$-cl$(h) = f$.*

Definition 3.3. *A soft subset h of a SDT-space $(f, \tilde{\delta})$ is called $\tilde{\delta}$-soft b-dense if $\tilde{\delta}$-sbcl$(h) = f$.*

Theorem 3.4. *Every $\tilde{\delta}$-soft b-dense set is $\tilde{\delta}$-soft dense.*

Proof. Let h be $\tilde{\delta}$-soft b-dense set. Then $\tilde{\delta}$-sbcl$(h) = f$. Since $\tilde{\delta}$-sbcl$(h)\tilde{\subseteq}\tilde{\delta}$-cl$(h)$, we have $\tilde{\delta}$-cl$(h) = f$ and so h is $\tilde{\delta}$-soft dense. □

Remark 3.5. The converse of the Theorem 3.4 need not be true that can be seen from the following example:

Example 3.6. Consider the SDT-space $(f, \tilde{\delta})$ and the Example 2.7, $g = f_9$ $\tilde{\tau} = \{f, \Phi, f_4, f_{10}\}$, $\tilde{\tau}_g = \{f, \Phi, f_1, f_7, f_{13}\}$. Then, we get soft ditopology over f as $\tilde{\delta} = \{f, \Phi, f_1, f_4, f_7, f_{10}, f_{13}\}$ and $\tilde{\delta}$-closed set are $\{f, \Phi, f_{12}, f_{14}, f_{11}, f_8, f_5\}$. $\tilde{\delta}$-soft b-unclosed sets are $\{f, \Phi, f_1, f_4, f_7, f_6, f_8, f_9, f_{12}, f_{10}, f_{13}\}$ and $\tilde{\delta}$-soft b-closed sets are $\{f, \Phi, f_1, f_2, f_7, f_3, f_8, f_5, f_{12}, f_{10}, f_{14}\}$. Take the soft subset $f_7 = \{(e_1, \{u_1\}), (e_2, \{u_2\})\}$ and $\tilde{\delta}$-cl$(f_7) = \{(e_1, \{u_1, u_2\})\} = f_3 \neq f$.
Thus f_7 is $\tilde{\delta}$-soft dense but not $\tilde{\delta}$-soft b-dense set.

Theorem 3.7. *If h is $\tilde{\delta}$-soft dense in $(f, \tilde{\delta})$, then h is $\tilde{\delta}$-soft b-unclosed in $(f, \tilde{\delta})$.*

Proof. Let h is $\tilde{\delta}$-simple dense in $(f, \tilde{\delta})$. Then $\tilde{\delta}$-cl$(h) = f$. Thus $\tilde{\delta}$-int$(\tilde{\delta}$-cl$(h)) = f$, which implies that $h\tilde{\subseteq}\tilde{\delta}$-int$(\tilde{\delta}$-cl$(h))\tilde{\subseteq}\tilde{\delta}$-int$(\tilde{\delta}$-cl$(h))\tilde{\cap}\tilde{\delta}$-cl$(\tilde{\delta}$-int$(h))$. Therefore h is $\tilde{\delta}$-simple b-unclosed in $(f, \tilde{\delta})$. □

Remark 3.8. In SDT- space $(f, \tilde{\delta})$, $\tilde{\delta}$-soft b-unclosed set h need not be soft dense set as shown in the following example:

Example 3.9. Consider the SDT-space $(f, \tilde{\delta})$ and the Example 2.7, $g = f_9$ $\tilde{\tau} = \{f, \Phi, f_4, f_{10}\}$, $\tilde{\tau}_g = \{f, \Phi, f_1, f_7, f_{13}\}$. Then, we get soft ditopology over f as $\tilde{\delta} = \{f, \Phi, f_1, f_4, f_7, f_{10}, f_{13}\}$ and $\tilde{\delta}$-closed set are $\{f, \Phi, f_{12}, f_{14}, f_{11}, f_8, f_5\}$. $\tilde{\delta}$-soft b-unclosed sets are $\{f, \Phi, f_1, f_4, f_7, f_6, f_8, f_9, f_{12}, f_{10}, f_{13}\}$ and $\tilde{\delta}$-soft b-closed sets are $\{f, \Phi, f_1, f_2, f_7, f_3, f_8, f_5, f_{12}, f_{10}, f_{14}\}$. The collection of $\tilde{\delta}$-soft dense sets of f are $\{f, \Phi, f_7, f_9, f_{13}$. Hence, $\{\Phi, f_1, f_4, f_7, f_6, f_8, f_9, f_{12}, f_{10}\}$ are $\tilde{\delta}$-soft b-unclosed sets but they are not soft dense set.

Definition 3.10. *Let $(f, \tilde{\delta})$ be a SDT-space and $h\tilde{\subseteq}f$.*
i.*h is $\tilde{\delta}$-simple regular unclosed set if $h = \tilde{\delta}$-int$(\tilde{\delta}$-cl$(h))$ and $\tilde{\delta}$-simple regular closed set if $h = \tilde{\delta}$-cl$(\tilde{\delta}$-int$(h))$.*
ii.*h is $\tilde{\delta}$-simple α unclosed set if $h\tilde{\subseteq}\tilde{\delta}$-int$(\tilde{\delta}$-cl$(\tilde{\delta}$-int$(h))$ and h is $\tilde{\delta}$-simple*

α closed set if $h\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($\tilde{\delta}$-cl($h$)).
iii.*h is $\tilde{\delta}$-simple pre unclosed set if $h\tilde{\subseteq}\tilde{\delta}$-int($\tilde{\delta}$-cl($h$)) and h is $\tilde{\delta}$-simple pre closed set if $h\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($h$)).*
iv.*h is $\tilde{\delta}$-simple semi unclosed set if $h\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($h$)) and h is $\tilde{\delta}$-simple semi closed set if $h\tilde{\subseteq}\tilde{\delta}$-int($\tilde{\delta}$-cl($h$)).*
v.*h is $\tilde{\delta}$-soft β-open ($\tilde{\delta} - s\beta - open$) set if $h\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($\tilde{\delta}$-cl($h$)) and h is $\tilde{\delta}$-soft β-closed ($\tilde{\delta} - s\beta - closed$) set if $\tilde{\delta}$-int($\tilde{\delta}$-cl($\tilde{\delta}$-int(h))$\tilde{\subseteq}h$.*

Theorem 3.11. *Let $(f, \tilde{\delta})$ be a SDT-space. Then, the $\tilde{\delta}$-soft sets have the following properties:*
i. *Every $\tilde{\delta}$-soft preunclosed set is $\tilde{\delta}$-soft β-unclosed.*
ii. *Every $\tilde{\delta}$-soft semi unclosed set is $\tilde{\delta}$-soft β-unclosed.*
iii. *Every $\tilde{\delta}$-soft α unclosed set is $\tilde{\delta}$-soft preunclosed.*

Proof. **i**. Let h be a $\tilde{\delta}$-simple preunclosed set. This implies, $h\tilde{\subseteq}\tilde{\delta}$-int($\tilde{\delta}$-cl($h$))$\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($\tilde{\delta}$-cl($h$)). Thus, h is $\tilde{\delta}$-simple β-unclosed set.
ii. Let h be a $\tilde{\delta}$-simple semi unclosed set. This implies, $h\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($h$))$\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($\tilde{\delta}$-cl($h$)). Thus, h is $\tilde{\delta}$-simple β-unclosed set.
iii. Let h be a $\tilde{\delta}$-simple α unclosed set. This implies, $h\tilde{\subseteq}\tilde{\delta}$-int($\tilde{\delta}$-cl($\tilde{\delta}$-int($h$))$\tilde{\subseteq}\tilde{\delta}$-int($\tilde{\delta}$-cl($h$)). Thus, h is $\tilde{\delta}$-simple preunclosed set. □

Remark 3.12. The converse of the above lemma is need not be true as seen in the following example:

Example 3.13. Consider the SDT-space $(f, \tilde{\delta})$ and the Example 2.7, $g = f_9$ $\tilde{\tau} = \{f, \Phi, f_4, f_{10}\}$, $\tilde{\tau}_g = \{f, \Phi, f_1, f_7, f_{13}\}$. Then, we get soft ditopology over f as $\tilde{\delta} = \{f, \Phi, f_1, f_4, f_7, f_{10}, f_{13}\}$ and $\tilde{\delta}$-closed set are $\{f, \Phi, f_{12}, f_{14}, f_{11}, f_8, f_5\}$.
i. f_6 is $\tilde{\delta}$-soft β unclosed but not $\tilde{\delta}$-soft preunclosed.
For proving the converse of the (**ii**) and (**iii**), we need to generate new SDT-spaces:

Example 3.14. Let us consider the soft subsets of f that are given in Example 2.7. $(f, \tilde{\delta})$ is a SDT-space, where $g = f_2$, $\tilde{\tau} = \{f, \Phi, f_1\}$, $\tilde{\tau}_g = \{f, \Phi, f_2\}$. Then $\tilde{\delta}$-soft unclosed sets are $\{f, \Phi, f_1, f_2, f_3\}$, $\tilde{\delta}$-soft closed sets are $\{f, \Phi, f_{12}, f_9, f_6\}$
ii. The soft subset f_4 is $\tilde{\delta}$-soft β-unclosed set but not $\tilde{\delta}$-soft semi unclosed.

Example 3.15. Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{e_1, e_2, e_3\}$ and $f = \{(e_1, \{u_1, u_2, u_3, u_4\}), (e_2, \{u_1, u_2, u_3, u_4\}), (e_3, \{u_1, u_2, u_3, u_4\})\}$. Then,

$\tilde{\tau}_1 = \{f, \Phi, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}, f_{13}, f_{14}, f_{15}\}$, $\tilde{\tau}_2 = \{f, \Phi\}$. Where,
$f_1 = \{(e_1, \{u_1\}), (e_2, \{u_2, u_3\}), (e_3, \{u_1, u_4\})\}$
$f_2 = \{(e_1, \{u_2, u_4\}), (e_2, \{u_1, u_3, u_4\}), (e_3, \{u_1, u_2, u_4\})\}$
$f_3 = \{(e_2, \{u_3\}), (e_3, \{u_4\})\}$
$f_4 = \{(e_1, \{u_1, u_2, u_4\}), (e_2, U), (e_3, U)\}$
$f_5 = \{(e_1, \{u_1, u_3\}), (e_2, \{u_2, u_4\}), (e_3, U)\}$
$f_6 = \{(e_1, \{u_1\}), (e_2, \{u_2\})\}$
$f_7 = \{(e_1, \{u_1, u_3\}), (e_2, \{u_2, u_3, u_4\}), (e_3, \{u_1, u_2, u_4\})\}$
$f_8 = \{(e_2, \{u_4\}), (e_3, \{u_2\})\}$
$f_9 = \{(e_1, U), (e_2, U), (e_3, \{u_1, u_2, u_3\})\}$
$f_{10} = \{(e_1, \{u_1, u_3\}), (e_2, \{u_2, u_3, u_4\}), (e_3, \{u_1, u_2\})\}$
$f_{11} = \{(e_1, \{u_2, u_3, u_4\}), (e_2, U), (e_3, \{u_1, u_2, u_3\})\}$
$f_{12} = \{(e_1, \{u_1\}), (e_2, \{u_2, u_3, u_4\}), (e_3, \{u_1, u_2, u_4\})\}$
$f_{13} = \{(e_1, \{u_1\}), (e_2, \{u_2, u_4\}), (e_3, \{u_2\})\}$
$f_{14} = \{(e_1, \{u_3, u_4\}), (e_2, \{u_1, u_2\})\}$
$f_{15} = \{(e_1, \{u_1\}), (e_2, \{u_2, u_3\}), (e_3, \{u_1\})\}$
Then the pair $\tilde{\delta} = (\tilde{\delta}, \tilde{\delta}_g)$ is a SDT-space over f.
Let $h \tilde{\subseteq} f$. Then, $h = \{(e_1, \{u_4\}), (e_2, \{u_1, u_2, u_3\}), (e_3, \{u_2, u_4\})\}$. $\tilde{\delta}$-int($\tilde{\delta}$-cl(h)) $= f$ and $\tilde{\delta}$-int($\tilde{\delta}$-cl($\tilde{\delta}$-int(h)) $= \Phi$. Hence, h is $\tilde{\delta}$-soft preunclosed set but not $\tilde{\delta}$-soft α-unclosed.

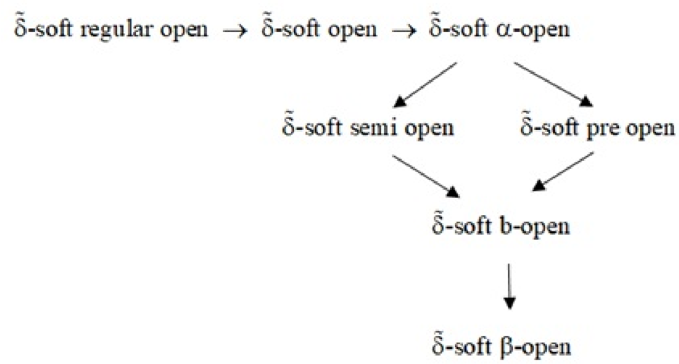
Theorem 3.16. *Let $(f, \tilde{\delta})$ be a SDT-space. Then,*
i. *Every $\tilde{\delta}$-soft preunclosed set is $\tilde{\delta}$-soft b-unclosed set.*
ii. *Every $\tilde{\delta}$-soft b-unclosed set is $\tilde{\delta}$-soft β-unclosed set.*
iii. *Every $\tilde{\delta}$-soft semi unclosed set is $\tilde{\delta}$-soft b-unclosed set.*

Proof. Let $(f, \tilde{\delta})$ be a SDT-space and $h \tilde{\subseteq} f$ and h is a $\tilde{\delta}$ simple preunclosed set. Then, $h \tilde{\subseteq} \tilde{\delta}$-int($\tilde{\delta}$-cl($h$))$\tilde{\subseteq}\tilde{\delta}$-int($\tilde{\delta}$-cl($h$))$\tilde{\cup}\tilde{\delta}$-int($h$)$\tilde{\subseteq}\tilde{\delta}$-int($\tilde{\delta}$-cl($h$))$\tilde{\cup}\tilde{\delta}$-cl($\tilde{\delta}$-int($h$)). Thus, (**i**) proved.
Let h be a $\tilde{\delta}$ simple b-unclosed set. Then, $h \tilde{\subseteq} \tilde{\delta}$-cl($\tilde{\delta}$-int($h$))$\tilde{\cup}\tilde{\delta}$-int($\tilde{\delta}$-cl($h$))$\tilde{\delta}$-cl($\tilde{\delta}$-int($\tilde{\delta}$-cl($h$)))$\tilde{\cup}\tilde{\delta}$-int($\tilde{\delta}$-cl($h$))$\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($\tilde{\delta}$-cl($h$))). Thus, (**ii**) proved.
Let h be a $\tilde{\delta}$ simple semi unclosed set. This implies, $h \tilde{\subseteq} \tilde{\delta}$-cl($\tilde{\delta}$-int($h$))$\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($h$))$\tilde{\delta}$-int($h$)$\tilde{\subseteq}\tilde{\delta}$-cl($\tilde{\delta}$-int($h$))$\tilde{\cup}\tilde{\delta}$-int($\tilde{\delta}$-cl($h$)). Thus, (**iii**) proved. □

Remark 3.17. The converse of the Theorem 3.15 is need not be true as seen in the following example:

Example 3.18. Let us consider the soft subsets of f that are given in Example 2.7. $(f, \tilde{\delta})$ is a SDT-space, where $g = f_2$, $\tilde{\tau} = \{f, \Phi, f_1\}$, $\tilde{\tau}_g = \{f, \Phi, f_2\}$. Then $\tilde{\delta}$-soft unclosed sets are $\{f, \Phi, f_1, f_2, f_3\}$, $\tilde{\delta}$-soft closed sets are $\{f, \Phi, f_{12}, f_9, f_6\}$
i. The soft set f_7 in f is $\tilde{\delta}$-soft b-unclosed set but not $\tilde{\delta}$-soft preunclosed set.
ii. The soft set f_5 in f is $\tilde{\delta}$-soft β-unclosed set but not $\tilde{\delta}$-soft b-unclosed set.
iii. The soft set f_4 in f is $\tilde{\delta}$-soft b-unclosed set but not $\tilde{\delta}$-soft semi unclosed set.
The accompanying comment gives the connection between $\tilde{\delta}$-soft sets.

Remark 3.19. The above discussions are summarized in the following diagrams:



4 An Approach to Generate New $\tilde{\delta}$- Soft Spaces in SDT-Spaces

In this section, I introduce $\tilde{\delta}$-soft b-extremally disconnected spaces and $\tilde{\delta}$-soft submaximal spaces with directly using new determined $\tilde{\delta}$- soft sets in Section 3. I obtain several characterizations of $\tilde{\delta}$- soft b-extremally disconnected spaces and $\tilde{\delta}$-soft b-submaximal spaces by utilizing classes of $\tilde{\delta}$- soft sets and rephrase them different from the probable Definition 4.2 and Definition 4.8.

The convenience and enthusiasm of this correspondence of $\tilde{\delta}$- soft sets will, obviously, be upgraded if there is a method for coming back from the changes to the $\tilde{\delta}$-soft sets, in other words, if there is an equation that describe $\tilde{\delta}$- soft spaces apart from given first definition. In closing this section, all the studies come to fruition and we take up a result which play a pivotal role in the characterization of $\tilde{\delta}$- soft b-extremally disconnected spaces and $\tilde{\delta}$-soft submaximal spaces.

Definition 4.1. *A SDT-space $(f, \tilde{\delta})$ is said to be $\tilde{\delta}$- soft extremally disconnected space if $\tilde{\delta}$ - closure of every $\tilde{\delta}$ - unclosed set of f is $\tilde{\delta}$- unclosed set in f.*

Definition 4.2. *A SDT-space $(f, \tilde{\delta})$ is said to be $\tilde{\delta}$- soft b-extremally disconnected space if $\tilde{\delta}$ - b-closure of every $\tilde{\delta}$ - b-unclosed set of f is $\tilde{\delta}$-b-unclosed set in f.*

Example 4.3. Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{e_1\}$ and $f = \{(e_1, \{u_1, u_2, u_3, u_4\})\}$. Then,
$f_1 = f$, $f_2 = \Phi$, $f_3 = \{(e_1, \{u_1\})\}$, $f_4 = \{(e_1, \{u_2\})\}$, $f_5 = \{(e_1, \{u_3\})\}$, $f_6 = \{(e_1, \{u_4\})\}$, $f_7 = \{(e_1, \{u_1, u_2\})\}$, $f_8 = \{(e_1, \{u_1, u_3\})\}$, $f_9 = \{(e_1, \{u_1, u_4\})\}$ $f_{10} = \{(e_1, \{u_2, u_3\})\}$, $f_{11} = \{(e_1, \{u_2, u_4\})\}$, $f_{12} = \{(e_1, \{u_3, u_4\})\}$, $f_{13} = \{(e_1, \{u_1, u_2, u_3\})\}$, $f_{14} = \{(e_1, \{u_1, , u_2, u_4\})\}$, $f_{15} = \{(e_1, \{u_2, u_3, u_4\})\}$, $f_{16} = \{(e_1, \{u_1, u_3, u_4\})\}$
Consider the SDT-space $(f, \tilde{\delta})$ where, g_{f3}, $\tilde{\tau} = \{f, \Phi, f_6, f_7, f_{14}\}$, $\tilde{\tau}_g = \{f, \Phi, f_3\}$. Then, $\tilde{\delta}$-unclosed sets are $\{f, \Phi, f_3, f_6, f_7, f_9, f_{14}\}$, $\tilde{\delta}$-soft b-unclosed sets are $\{f, \Phi, f_3, f_4, f_6, f_7, f_8, f_9, f_{12}, f_{13}, f_{14}, f_{16}\}$.
Then $\tilde{\delta}$-soft b-closure of every $\tilde{\delta}$-soft b-unclosed set of f is $\tilde{\delta}$-soft b-unclosed set in f. Hence is $\tilde{\delta}$-soft b-extremally disconnected space.

Remark 4.4. Every $\tilde{\delta}$-soft extremally disconnected space is $\tilde{\delta}$-soft b-extremally detached space yet the opposite need not be fulfilled as appeared in the accompanying model:

Example 4.5. Consider the SDT-space $(f, \tilde{\delta})$ given in Example 4.3. Here, $\tilde{\delta}$-cl$(f_6) = \tilde{\delta}$-cl$(\{(e_1, \{u_4\})\}) = f_{12} = \{(e_1, \{u_3, u_4\})\}$ which is not a $\tilde{\delta}$-unclosed set. Therefore, $(f, \tilde{\delta})$ is a $\tilde{\delta}$-soft b-extremally disconnected space but not $\tilde{\delta}$-soft extremally disconnected space.

Theorem 4.6. *Let $(f, \tilde{\delta})$ be SDT-space and $h, k \tilde{\subseteq} f$. Then the following statements are equivalent:*

i.*f is $\tilde{\delta}$-soft b-extremally disconnected space.*
ii.*$\tilde{\delta}$-sbint(h) is $\tilde{\delta}$-soft b-closed set for each $\tilde{\delta}$-soft b-closed set h of f.*
iii.*$\tilde{\delta}$-sbcl$(\tilde{\delta}$-sbcl$(h))^{\tilde{c}} = (\tilde{\delta}$-sbcl$(h))^{\tilde{c}}$ for each $\tilde{\delta}$-soft b-closed set h of f.*
iv.*$k = \tilde{\delta}$-sbcl$(\tilde{\delta}$-sbcl$(h))^{\tilde{c}}$ implies $\tilde{\delta}$-sbcl$\tilde{\delta}$-sbcl$(k) = (\tilde{\delta}$-sbcl$(h))^{\tilde{c}}$ for each pair of $\tilde{\delta}$-soft b-unclosed set h and k of f.*

Proof. (**i**) $\Rightarrow$ (**ii**) Let h be a $\tilde{\delta}$-soft b-closed set of f. Then, $h^{\tilde{c}}$ is $\tilde{\delta}$-soft b-unclosed. Since f is $\tilde{\delta}$-soft b-extremally disconnected space, $\tilde{\delta}$-sbcl$(h^{\tilde{c}})$ is $\tilde{\delta}$-soft b-unclosed set. But, $\tilde{\delta}$-sbcl$(h^{\tilde{c}}) = (\tilde{\delta}$-sbint$(h))^{\tilde{c}}$. Therefore $\tilde{\delta}$-sbint is $\tilde{\delta}$-soft b-closed set.
(**ii**) $\Rightarrow$ (**iii**) Suppose that h is $\tilde{\delta}$-soft b-unclosed set h of f. Then, $\tilde{\delta}$-sbcl$(\tilde{\delta}$-sbcl$(h))^{\tilde{c}} = \tilde{\delta}$-sbcl$(\tilde{\delta}$-sbint$(h^{\tilde{c}}))$ By assumption, $\tilde{\delta}$-sbint$(h^{\tilde{c}})$ is a $\tilde{\delta}$-soft b-closed set. So $\tilde{\delta}$-sbcl$(\tilde{\delta}$-sbint$(h^{\tilde{c}})) = \tilde{\delta}$-sbint$(h^{\tilde{c}}) = (\tilde{\delta}$-sbint$(h))^{\tilde{c}}$.
(**iii**) $\Rightarrow$ (**iv**) Let h and k be $\tilde{\delta}$-soft b-unclosed set of f. We get $k = (\tilde{\delta}$-sbcl$(h))^{\tilde{c}}$. By assumption, $\tilde{\delta}$-sbcl$(k) = \tilde{\delta}$-sbcl$(\tilde{\delta}$-sbcl$(h))^{\tilde{c}} = (\tilde{\delta}$-sbcl$(h))^{\tilde{c}}$.
(**iv**) $\Rightarrow$ (**i**) Let h be a $\tilde{\delta}$-soft b-unclosed set of f. Let $k = (\tilde{\delta}$-sbcl$(h))^{\tilde{c}}$. From the assumption, we obtain that $\tilde{\delta}$-sbcl$(k) = (\tilde{\delta}$-sbcl$(h))^{\tilde{c}}$. So $(\tilde{\delta}$-sbcl$(k))^{h^{\tilde{c}}} = \tilde{\delta}$-sbcl$(h)$. Hence, $\tilde{\delta}$-sbint$(k^{\tilde{c}}) = \tilde{\delta}$-sbcl$(h)$. Thus, $\tilde{\delta}$-sbcl(h) is $\tilde{\delta}$-soft b-unclosed set of f. Then, f is $\tilde{\delta}$-soft b-extremally disconnected space. $\square$

Definition 4.7. *A SDT-space $(f, \tilde{\delta})$ is said to be $\tilde{\delta}$-soft submaximal if every $\tilde{\delta}$-soft dense subset is $\tilde{\delta}$- unclosed set in f.*

Definition 4.8. *A SDT-space $(f, \tilde{\delta})$ is said to be $\tilde{\delta}$- b-soft submaximal if every $\tilde{\delta}$-soft dense subset is $\tilde{\delta}$-soft-b-unclosed set in f.*

Example 4.9. Let us consider the SDT-space $(f, \tilde{\delta})$ and the Example 2.7. $g = f_3$, $\tilde{\tau} = \{f, \Phi, f_1, f_7\}$, $\tilde{\tau}_g = \{f, \Phi, f_3\}$. Then, we get soft ditopology over f as $\tilde{\delta} = \{f, \Phi, f_1, f_7, f_8, f_{13}\}$ and $\tilde{\delta}$-closed set are $\{f, \Phi, f_5, f_6, f_{11}, f_{12}\}$. Then the collection of $\tilde{\delta}$-soft b-unclosed sets is $\{f, \Phi, f_1, f_3, f_7, f_8, f_9, f_{12}, f_{13}, f_{14}\}$. Also, the collection of $\tilde{\delta}$-soft dense sets is $\{f, \Phi, f_1, f_3, f_7, f_8, f_9, f_{12}, f_{13}, f_{14}\}$. Hence, all $\tilde{\delta}$-soft dense sets are $\tilde{\delta}$-soft b-unclosed set and so $(f, \tilde{\delta})$ is a $(f, \tilde{\delta})$-soft b-submaximal space.

Theorem 4.10. *Every $\tilde{\delta}$-soft submaximal space is $\tilde{\delta}$-soft b-submaximal.*

Proof. Let $(f, \tilde{\delta})$ be a $\tilde{\delta}$-soft submaximal space and h be a $\tilde{\delta}$-soft dense subset of f. Then h is $\tilde{\delta}$- unclosed set in f. It is proved that every $\tilde{\delta}$-

unclosed set is $\tilde{\delta}$-soft b-unclosed set and so h is $\tilde{\delta}$-soft b-unclosed set. Therefore $(f, \tilde{\delta})$ is $\tilde{\delta}$-soft b-submaximal.
The reverse implication of Theorem 4.10 is not true that can be seen in the following example: □

Example 4.11. Let $(f, \tilde{\delta})$ be a $\tilde{\delta}$-soft b-submaximal space that is given in Example 4.9. Here the $\tilde{\delta}$-soft dense set f_8 is $\tilde{\delta}$-soft b-unclosed set but not $\tilde{\delta}$- unclosed set in f.
Therefore $\tilde{\delta}$-soft b-submaximal space is not $\tilde{\delta}$-soft submaximal space.

Theorem 4.12. *Every SDT - space is a $\tilde{\delta}$-soft b-submaximal space.*

Proof. Let $(f, \tilde{\delta})$ be a SDT - space and h be any $\tilde{\delta}$-soft dense set of f. Then, by Definition 3.9, h is $\tilde{\delta}$-soft b-unclosed set. Therefore $(f, \tilde{\delta})$ is a $\tilde{\delta}$-soft b-submaximal space. □

5 Conclusion

In this study I aim to discuss how soft set theory can be used for developing SDT-spaces by using $\tilde{\delta}$-b-unclosed set, $\tilde{\delta}$-b-closed set, $\tilde{\delta}$-b-dense set, $\tilde{\delta}$-regular- unclosed set, $\tilde{\delta}$-preunclosed set, $\tilde{\delta}$-semi unclosed set, $\tilde{\delta}$-α-unclosed set and $\tilde{\delta}$ -β- unclosed set which are first mentioned in this work. Moreover, $\tilde{\delta}$- soft b-extremally disconnected spaces and $\tilde{\delta}$-soft submaximal spaces are introduced with directly using new determined $\tilde{\delta}$-soft sets. Several characterizations of $\tilde{\delta}$-soft b-extremally disconnected spaces and $\tilde{\delta}$-soft b-submaximal spaces are developed by utilizing classes of $\tilde{\delta}$-soft sets. In shutting this examination, every one of the investigations happen as expected and we take up an outcome which assumes a significant job in the portrayal of $\tilde{\delta}$-soft b-extremally disconnected spaces and $\tilde{\delta}$-soft submaximal spaces.

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