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Original Research Paper

Using Malmquist Productivity Index in a Two-Stage Sustainable Supply Chain

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Abstract. Data envelopment analysis (DEA) is used as a tool to evaluate a set of decision-making units (DMUs) forming a sustainable supply chain (SSC). Furthermore, the Malmquist Productivity Index (MPI) is used to determine the level of progress or regression in the DMUs. In this paper, we calculate the MPI in a sustainable supply chain using non-radial DEA models. Therefore, a non-radial Enhanced Russell model in two-stage supply chain is proposed to determine the efficiency of each DMU by considering economic, social and environmental sustainability factors. Thus, in this paper, MPI is calculated in a sustainable supply chain using the non-radial Enhanced Russell Measure (ERM). The proposed model considers the priority of decreasing inputs and increasing outputs by the decision-maker. Moreover, our applied study involves an evaluation of sustainable supply chains in Iranian oil refineries.

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1. Introduction

Data envelopment analysis was first introduced based on an idea proposed by Farrell (1957), and later developed by Charnes et al. [1] in the form of the CCR model. Later on, Banker et al. [2] proposed the topic of variable returns to scale (VRS) technology, and introduced the BCC model. In recent years, DEA has been used as a powerful mathematical programming-based tool for evaluation of organizational performance. Performance evaluation is of great importance in the majority of organizations, as it determines the strengths and weaknesses, as well as the progress and regression, of the decision-making units (DMUs). Nowadays, managers are paying serious attention to supply chain evaluation with an emphasis on the conditions of sustainability. Therefore, tools such as DEA (Data Envelopment Analysis) play a significant role in the ranking, benchmarking of units, as well as determination of their progress and regression.

DEA has various applications. in this regard, we refer the study by Liu et al. [3], which provides a review of DEA applications in literature from 1978 to 2010. In a similar study Wanke and Barros [4] used a two-stage approach to evaluate 59 banks in Brazil. In the same year, Titko and Jureviciene [5] presented a VRS-based DEA model for evaluation of 16 banks in Tatvian. Sueyoshi et al. [6] reviewed the literature from 1980 to 2010 based on factors associated with energy and the environment. Furthermore, MPI was used in the study by Fernandes et al. [7] to predict the financial risks endangering the European banks.

In the competitive market of today, manufacturers have to deliver their products with the highest quality, in the fastest time and with the lowest cost possible in order to remain in the scene. Therefore, it is fundamental for them to have a strong and efficient supply chain.

Generally, the set of transportation, service and production companies and institutes that, either directly or indirectly, play a role in meeting the demands of a customer are called a supply chain. Furthermore, a supply chain usually includes the suppliers of raw material, capital suppliers (banks and financial institutions), manufacturers, distribution companies, wholesalers, retailers and customers.

In classic data envelopment analysis, the supply chain is considered as a black box, in which only inputs and outputs are used for efficiency evaluation. Therefore, intermediate products are ignored completely. In supply chain management, efficiency evaluation has a significant and critical role in reaching the two objectives of reducing costs and increasing profits. Moreover, DEA is used as a tool to help with managerial decision makings in efficiency measurement of supply chains. Since every independent decision-making system in any member of the supply chain only tries to maximize its technical efficiency, and ignores the other members and the entire supply chain, it is therefore essential to use network-based models in order to pay attention to the whole supply chain, or in simpler words, look inside the black box. In network-based models of the supply chain, which include the suppliers, the manufacturers and the distributers, relationships are in a way that for instance, the outputs of the supplier are considered inputs for the manufacturer.

In the study by Yu et al. [8], information sharing and cross-efficiency were used to evaluate weight flexibility. Supply chain performance and its indicator were discussed by Agrell and Hatami-Marbini [9] through a study of supply chains in the Indian automotive industry. Zhang et al. [10] used a two-stage DEA model to evaluate the supply chains involved in oil import in China. Azadi et al. [11] introduced two different DEA models to benchmark two-stage supply chains in 24 public transportation service providers in Tehran. Mirhedayatian et al. [12] used a dual-role NDEA model for evaluation of green supply chain management (GSCM). Grigoroudis et al. [13] presented a Ratio DEA (RDEA) algorithm for evaluation of supply chain networks, this algorithm was based on the use of MITP models. In a reprint of their previous article, Moreover, Balfaqih et al. [14] reviewed the techniques, methods and criteria presented for supply chain performance measurement from

1998-2015. Finally, Babazadeh et al. [15] used a hybrid model in data envelopment analysis to evaluate the biodiesel supply chain network.

In a sustainable supply chain, organizations must consider profitability and competitive advantage on one hand, and waste elimination and minimization on the other. Sustainable supply chain management involves an assessment of socioeconomic and environmental factors associated with the supply chain. In this regard, organizations must always pay attention to factors associated with quality social service provision, such as respect for human rights, education, health, gender equality, occupational safety and workplace quality. In a sustainable environmental system, it is necessary to maintain the sustainable resources, and be frugal in consumption of non-renewable energies. It is also important to keep a constant watch on the consumption of energy and water. In order to have a sustainable economic system, goods and services must be constantly produced with high quality. We must also continuously try to improve the quality of our products in order to meet all customer needs. Furthermore, we can take a huge step toward customer satisfaction by providing long-term guarantees. In many organizations, managers make their decisions for reaching organizational goals based on the progress or regression of the decision-making units. Therefore, organizations that form a supply chain are quite important in terms of the two-stage processes they have and the sustainability of economic, environmental and social conditions.

Yakovteva et al. [16] studied three dimensions of supply chain sustainability for potato production companies in the UK. Moreover, the BSC method was used by Shafiee et al. [17] to evaluate network-based supply chains, as well as the food supply chain in Iran. Later on, Nikfarjam et al. [18] used specific forms of a hybrid model to assess a supply chain of 7 DMUs. In the study conducted by Ding et al. [19], supply chain sustainability was evaluated based on environmental factors in a case study relating the impact of business on China's environment. Motevati Haghighi et al. [20] used the DEA-BSC model to assess network-based models of supply chain and do research on 40 plastic recycling companies. Farzipoor Farzipoor saen et al. [21] engaged in a study of two-stage network DEA with negative data extracted from radial and non-radial models. Their case study involved 29 medical equipment manufacturing companies. Later, Izadikhah and Farzipoor Farzipoor saen [22] used a two-stage network model to calculate desirable and undesirable efficiency scores, and then arrive at the overall efficiency. In the study by Boudaghi et al. [23], discriminant DEA models were used to predict the members of a sustainable supply chain.

The Malmquist Productivity Index (MPI) is an indicator of changes in units in two consecutive periods of time. This index is defined as a ratio of differences in the unit's efficiency scores between two different points in time. Among advantages to the MPI is the fact that we can divide it into two parts, namely changes in the efficiency frontier and changes in the efficiency scores at two different times.

In the work by B. Walheer [24], we can observe the use of the Cost Malmquist Productivity Index for assessment of joint and output-specific inputs. Z. Li [25] engaged in prediction of financial risks in 742 Chinese companies using the MPI. Finally, D. Fernández [26] used the MPI to evaluate 34 Air Separation Units (ASUs) in Europe and Asia.

Considering the significance of supply chain evaluation in Iranian oil refineries, the current study uses the Malmquist productivity index (MPI) for this purpose in a two-stage network consisting of suppliers and producers.

This process of evaluating the decision-making units (DMUs) through the MPI in a two-stage network is important for two main reasons:

1. Non-radial models can be used in our DMU evaluation in a way that a ratio of the mean reduction in inputs to the mean increase in outputs can be used as a criterion for calculating the MPI in the case of each DMU.

2. Use of two-stage supply chain networks. in this regard, since this study focuses on suppliers and producers, there is a crucial need to consider the intermediate vectors. Furthermore, it is extremely important to take environmental, social and economic factors into account when calculating the Malmquist productivity index.

The rest of the current article is structured as will follow:

In section two, we present the concepts associated with a sustainable supply chain and provide a brief introduction to data envelopment analysis. Next, after introducing two-stage network processes with undesirable outputs, we propose our approach to calculate the MPI in each separate stage of the supply chain, as well as the overall supply chain, in section three. An applied study is conducted in section four, where the MPI is calculated for 9 supply chains in Iranian oil refineries.

2. Basic Concepts

In this section, we first briefly define the basic concepts of supply chain management and supply chain sustainability, and then describe the non-radial Enhanced Russell Measure (ERM) in data envelopment analysis.

2.1 Sustainable supply chain

Generally, a supply chain is made up of suppliers, manufacturers and consumers, and management of any of these components can lead to the efficiency of the supply chain. However, suppliers alone can play a significant role in preservation of natural factors, and manufacturers influence the protection of environmental and economic factors. Finally, as an important part of the supply chain, consumers can manage all three of the environmental, social and economic factors. In this regard, supply chain sustainability is an essential factor that needs to be institutionalized in society and the industry. For instance, in a bottled water manufacturing company, the main product is water supplied from springs, but water is not the only factor in the process of selling bottled water. To produce a bottle of mineral water, we also need a plastic bottle, an appropriate lid and a label. In addition, the mineral water has to be purified first, and go through various stages of quality control and health standardization. Usually, bottled water manufacturers do not produce the bottles themselves. Therefore, they need to buy the bottles from another company and transport them to their manufacturing plant. The companies have to use trucks for this purpose. Trucks need drivers and fuel. Moreover, the truck, its driver and the goods inside all need to be insured. Emptying the bottles in the factory would require special unloading machines and a number of manual laborers, who all must be insured as well. Management of this chain from supply of raw materials to the selling of final products is referred to as "supply chain management".

In 2017, Badiezadeh et al, [27] calculated the overall efficiency using a combination of optimistic efficiency and pessimistic efficiency, and then evaluated the supply chains in 9 Iranian tomato paste manufacturing companies. Izadikhad et al. (2016) [21] presented a two-stage supply chain using negative data, and then investigated 9 pharmaceutical industries in Iran. Finally, Boudaghi et al. (2018) [23] used a discriminant DEA model to evaluate sustainable supply chains. they demonstrated the application of their suggested approach in 20 car manufacturing companies belonging to Iran Khodro (IKCO) in Iran.

Sustainability is humans' need to earn money from nature's resources, rather than their own resources in life. In order to achieve sustainability, companies try to manufacture products that are not harmful to the environment and conform to social standards.

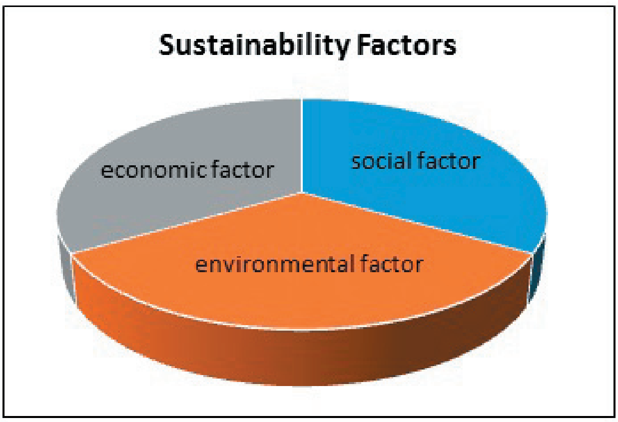

Figure 1 shows the different aspects of a sustainable supply chain.

Sustainable supply chain management is rooted in sustainability and involves an extensive approach to supply chain management. Sustainability in a supply chain means to direct the supply chain toward paying attention to economic, environmental and social aspects, and resolving the issues existing around these aspects in traditional supply chains.

In the social aspect of sustainable supply chain, the main focus is on competitive criteria. The economic aspect involves the concepts of cost, productivity and so on, and finally, the environmental aspect revolves around waste production levels.

2.2 Data envelopment analysis in the enhanced russel measure (ERM)

Data envelopment analysis (DEA), first introduced by Charnes et al. in 1978 through the CCR model and later defined by Banker et al. (1984) using the BCC model, created a certain attitude in radial models with the aim to reduce inputs or increase outputs. Later on, the Enhanced Russell Measure (ERM) was presented by Pastor et al. in 1999, and Tone proposed the Slack Based Measure (SBM) model in 1997. If we consider a DMU_j with the input vector $X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$, including m input components, and the output vector $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$, comprising s output components, we can provide production possibility set based on consumption of m inputs and production of s outputs.

In this regard, Pastor et al. (1999) [28] proposed the Enhanced Russell Measure (ERM) to evaluate DMU_o under assumption of variable returns to scale as follows:

$$
\begin{aligned}
Min \quad & \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r} \\
s.t. \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \leqslant \theta_i x_{io}, \qquad i = 1, \ldots, m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \geqslant \varphi_r y_{ro}, \qquad r = 1, \ldots, s, \\
& \sum_{j=1}^{n}\lambda_j = 1, \\
& \theta_i \leqslant 1, \qquad i = 1, \ldots, m, \\
& \varphi_r \geqslant 1, \qquad r = 1, \ldots, s, \\
& \lambda_j \geqslant 0, \qquad j = 1, \ldots, n.
\end{aligned} \tag{1}
$$

θ_i and φ_r in model (1) denote the values for the radial decrease in the inputs and the radial increase in the outputs, respectively. The objective function of this model shows the ratio of the mean radial efficiency of inputs to the mean radial efficiency of outputs for DMU_o (the unit under evaluation).

Model (1) is presented with the aim to reduce the average of input coefficients and increase the average of output coefficients with consideration to the production possibility conditions for DMU_o, which is a given decision-making unit under evaluation. Restrictions $\theta_i \leqslant 1$ and $\varphi_r \geqslant 1$ in model (1) make it possible to project the inefficient units on the efficiency frontier by reducing and increasing the input and output vectors, respectively. The constraints in model (1) are linear, but its objective function is in the form of a fraction

Definition 2.2.1. *DMU_o is efficient in model (1) if $R^*{}_o = 1$, where $R^*{}_o$ is the optimal solution of model (1).*

3. Malmquist Productivity Index in a Two-Stage Supply Chain

Numerical indexes are important in measurement of productivity and its changes. The Malmquist Productivity index (MPI) is one of the indices with such characteristics. The MPI is used to determine the amount of change in the productivity of all production factors. Among the interesting attributes of this index is that calculating it does not require any information on the prices of production factors and products, which is often difficult or impossible to collect. This index does not aim to maximize profits or minimize costs, but rather tries to make changes in the technical efficiency, and influence the technology through this change. The mathematical model of the Malmquist index is defined based on a distance function, in which the productivity changes in all production factors between two data points are measured based on the distance of each point to a common technology. Distance functions can be used in measurement and analysis of efficiency and productivity. The distance function can be approached from two aspects: one, based on inputs or production factors, which is known as the input distance function and focuses on minimum consumption of production factors, and two, based on outputs, which is known as the output distance function and focuses on maximum production of outputs. Interestingly, the attribute of returns to scale in production is of great importance in the Malmquist productivity index.

In the majority of organizations, we can design and then evaluate a network based on the vectors of inputs, outputs and intermediate measures. Although, the type of data in these vectors, such as fuzzy, random and binary, is of great consequence. However, after disregarding imprecise data, certain vectors in the DEA network could still be either desirable or undesirable. For instance, in the example of bottled water manufacturing companies, lack of control on the returning of plastic bottles into the production cycle could be considered an undesirable environmental factor. Furthermore, incorrect use of water resources can be an economic, social and environmental factor in the supply chain. Similarly, the culture of using bottled water would improve public health levels in society, and thereby, influence the social factors. In the following, a two-stage network process with undesirable outputs is illustrated for a two-stage supply chain.

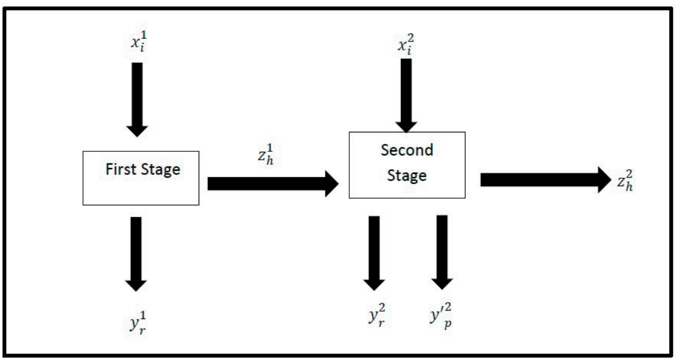

Figure 2. Two-stage network process with undesirable outputs.

Input, output and intermediate vectors are defined for a two-stage network as follows:

x_i^1 : Input of first stage $i \in I_1$

y_r^1: Desirable output of first stage $r \in R_1$

x_i^2 : Input of second stage $i \in I_2$

y_r^2: Desirable output of second stage $r \in R_2$

z_h^2: Output of second stage h $\in S_2$

y'^2_p: Undesirable output of second stage $p \in P$

z_h^1: Output of first stage and input of second stage h $\in S_1$

In many industrial factories, the pollution and waste resulting from production can be an undesirable output, which we should try to reduce. In other words, these outputs act as inputs, and the lesser their amount, the higher the efficiency value will be. Needless to say, in evaluation of a set of DMUs forming a supply chain, data envelopment analysis can be used as a tool to calculate the level of progress and regression. The main objective in this section is to calculate the MPI in a two-stage supply chain using non-radial models. To this end, the MPI is first calculated in each individual stage of the supply chain, and then, a model is proposed to calculate the overall MPI of the two-stage supply chain.

3.1 MPI in individual supply Chain stages

In general, undesirable outputs in a supply chain are considered inputs in the process. Performance evaluation of organizations with multi-stage processes within a specific time period is not a very accurate criterion for the overall evaluation. Therefore, evaluating the units in different time intervals would specify the level of progress or regression.

In this section, the MPI is calculated in a two-stage supply chain using the non-radial Enhanced Russell Measure (ERM). In this regard, the MPI is first calculated in each stage of a sustainable supply chain separately, and then the overall MPI is determined in the two-stage process. Note that during MPI calculation in first stage, vector X is the input vector and Z denotes the output vector, and in the second stage, Z represents the vector of inputs and Y is the output vector.

In the following, we present the enhanced Russell model for first stage at the time of t for DMU$_o$ and time of t+1 for another DMUs.

$$
\begin{aligned}
RL_o^{1^*} = Min \quad & \frac{\frac{1}{m_1}(\sum_{i\in I_1} v_{1i}\theta_{1i})}{\frac{1}{s_1}}(\sum_{r\in R_1} u_{1r}\varphi_{1r}) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j^1 {x_{ij}^1}^t \leqslant \theta_i {x_{io}^1}^{t+1}, & i \in I_1, \\
& \sum_{j=1}^{n} \lambda_j^1 {y_{rj}^1}^t \geqslant \varphi_r {y_{ro}^1}^{t+1}, & r \in R_1, \\
& \sum_{j=1}^{n} \lambda_j^1 = 1, \\
& \theta_i \leqslant 1, & i \in I_1, \\
& \varphi_r \geqslant 1, & r \in R_1, \\
& \lambda_j^1 \geqslant 0, \quad j = 1, \ldots, n,
\end{aligned}
\tag{2}
$$

where $|I_1| = m_1$ and $|R_1| = S_1$.

Note that DMU$_o$ is considered at the time of t+1, and the time t is considered for the other DMUs.

In model (2), θ_i and φ_r represent the radial decreases in the inputs and outputs of the first stage, respectively. In addition, ${RL_o}^{1^*}$ denotes the efficiency score obtained from the Russel model in first stage.

Furthermore, the restrictions are linear and the objective function has a fraction and non-linear form. thus, we can illustrate the objective function as linear[29]. Moreover, in this model, vectors ${x_{ij}^1}^t$ and ${y_{rj}^1}^t$ are the input and output vectors, respectively. Also v_{1i} and $u_{1r}$are the priorities that the decision maker (DM) proposes to reduce inputs and increase outputs, respectively. After solving model (2), the optimal value of the objective function is defined as follows:

$$RL_o^{1^*} = M^t({x_o^1}^{t+1}, {y_o^1}^{t+1}) \tag{3}$$

Now, the MPI is calculated for the first stage as follows:

$$MQ_1 = \left(\frac{M^t(x_o^{1\ t+1}, y_o^{1\ t+1}).M^{t+1}(x_o^{1\ t+1}, y_o^{1\ t+1})}{M^t(x_o^{1\ t}, y_o^{1\ t}).M^{t+1}(x_o^{1\ t}, y_o^{1\ t})}\right)^{\frac{1}{2}} \tag{4}$$

In equation (4), MQ_1 denotes the Malmquist productivity index in the first stage.
In a similar way, the enhanced Russell model for second stage at the time of t will be as follows:

$$RL_o{}^2 = Min \frac{\frac{1}{m_2+s_3}(\alpha_1 \sum_{i\in I_2} v_{2i}\delta_i + \beta_1 \sum_{p\in P} v_{2p}\gamma_p)}{\frac{1}{s_2+K_2}(\alpha_2 \sum_{r\in R_2} u_{2r}\rho_r + \beta_2 \sum_{h\in H_2} u_{2hh})}$$

$$\text{S.t.} \sum_{j=1}^{n} \lambda_j^2 x_{ij}^{2\ t} \le \delta_i x_{io}^{2\ t+1} \qquad i \in I_2, \qquad (5-1)$$

$$\sum_{j=1}^{n} \lambda_j^2 y_{rj}^{2\ t} \ge \rho_r y_{ro}^{2\ t+1} \qquad r \in R_2 \qquad (5-2)$$

$$\sum_{j=1}^{n} \lambda_j^2 y'^{2\ t}_{pj} \le \gamma_p\, y'^{2\ t+1}_{po} \qquad p \in P, \qquad (5-3)$$

$$\sum_{j=1}^{n} \lambda_j^2 z_{hj}^{2\ t} \ge {}_h z_{ho}^{2\ t+1} \qquad h \in H_2,$$

$\delta_i \le 1,$ $\quad i \in R_3,$
$\gamma_p \le 1,$ $\quad p \in P,$
$\rho_r \ge 1,$ $\quad r \in R_2,$
$\psi_h \ge 1,$ $\quad h \in H_2,$
$\gamma_p \ge 1,$ $\quad r \in R_2,$
$\lambda_j^2 \ge 0,$ $\quad j = 1, \ldots, n.$

$$\sum_{j=1}^{n} \lambda_j^2 = 1. \tag{5}$$

where $|I_2| = m_2$, $|R_2| = s_2$, $|P| = s_3$ and $|H_2| = K_2$.

Constraints of Model (5) have a linear form though their objective function is fractional and nonlinear, but they can be transformed into a linear form by changing the variable as suggested by Charnes and Cooper (Chames et al., 1978).

In the model (5), vectors ${x_{ij}^{2}}^{t}$, ${z_{hj}^{2}}^{t}$, ${y_{rj}^{2}}^{t}$ and ${y'^{2}_{pj}}^{t}$ are input and output vectors, respectively.

Moreover, δ_i and γ_p are the ratios of reductions in the inputs and ρ_r and ψ_h are the ratios of increases in the outputs in second stage.

Also v_{2i} and $v_{2p}$are input priorities and u_{2r} and $u_{2h}$are output priorities.

In regard to the constraint $(5-3)$, we can point out that in the evaluation of units, our aim is to reduce the inputs and increase the outputs, so that the unit reaches optimal performance. However, we must note that organizations are not always looking to maximize the outputs and minimize the inputs, as the inputs and outputs could be desirable or undesirable. Therefore, to improve the performance, we must increase the undesirable inputs and reduce the desirable outputs. In the objective function of model (5), the values of the parameters α_1, α_2, β_1 and β_2 are specified based on the manager's opinion. If the manager wants the inputs to get reduced more than the undesirable outputs, we set $\beta < \alpha$, and otherwise, we set $\beta > \alpha$; in addition, if the reduction was to be the same in the inputs and the undesirable outputs, we let $\beta = \alpha$.

$$RL_o{}^{2*} = M^t\left(x_o^{2t+1},\ y_o^{2t+1}\ ,\ z_o^{2t+1}, y'^{2t+1}_o\right) \tag{6}$$

In equation (6), $RL_o{}^{2*}$ is the optimal solution of model (5). Thus, the Malmquist Productivity Index for DMU$_o$ in the second stage is calculated as follows:

$$MQ_2 = \left(\frac{M^t\left(x_o^{2t+1},\ y_o^{2t+1}\ ,\ z_o^{2t+1},\ y_o'^{2t+1}\right).M^{t+1}\left(x_o^{2t+1},\ y_o^{2t+1}\ ,\ z_o^{2t+1},\ y_o'^{2t+1}\right)}{M^t\left(x_o^{2t},\ y_o^{2t}\ ,\ z_o^{2t},\ y_o'^{2t}\right).M^{t+1}\left(x_o^{2t},\ y_o^{2t}\ ,\ z_o^{2t},\ y_o'^{2t}\right)}\right)^{\frac{1}{2}} \tag{7}$$

Therefore, equations (4) and (7) calculate the MPI for DMU$_o$ in first and second stages, respectively. However, calculating the overall MPI for both stages is of great importance, which will be discussed in Section 3.2.

3.2 Overall MPI of the supply Chain network

In a two-stage supply chain process, we consider the input and output vectors (both desirable and undesirable) in the ERM aiming to reduce the inputs and undesirable outputs and increase the desirable outputs. There are intermediate vectors in a two-stage process, and thus, the output vector of first stage is bigger than the input vector of second stage. Figure 2 shows the two-stage process in a supply chain, and the following proposes the corresponding non-radial enhanced Russell model for evaluation of overall efficiency in DMU_o at the time of t+1, other units are evaluated at the time of t.

$$RLT^{o*} = Min \frac{\frac{1}{m_1+m_2+K_1+s_3}(\alpha_1 \sum_{i\in I_1} v_{1i}\theta_i + \alpha_2 \sum_{i\in I_2} v_{2i}\delta_i + \alpha_3 \sum_{h\in H_1} v_{1h}\eta_h + \alpha_4 \sum_{p\in P} v_{2p}\gamma_p)}{\frac{1}{s_1+K_2+s_2}(\beta_1 \sum_{r\in R_1} u_{1r}\varphi_r + \beta_2 \sum_{r\in R_2} u_{2r}\rho_r + \beta_3 \sum_{h\in H_2} u_{2hh})}$$

S.t. Stage1

$$\sum_{j=1}^{n} \lambda_j^1 {x_{ij}^1}^t \leq \theta_i \ \ {x_{io}^1}^{t+1} \qquad i \in I_1,$$

$$\sum_{j=1}^{n} \lambda_j^1 {y_{rj}^1}^t \geq \varphi_r {y_{ro}^1}^{t+1} \qquad r \in R_1,$$

Stage2

$$\sum_{j=1}^{n} \lambda_j^2 {y_{rj}^2}^t \geq \rho_r {y_{ro}^2}^{t+1} \qquad r \in R_2,$$

$$\sum_{j=1}^{n} \lambda_j^2 {x_{ij}^2}^t \leq \delta_i {x_{io}^2}^{t+1} \qquad i \in I_2\ ,$$

$$\sum_{j=1}^{n} \lambda_j^2 {z_{hj}^2}^t \geq \psi_h {z_{ho}^2}^{t+1} \qquad h \in H_2,$$

$$\sum_{j=1}^{n} \lambda_j^2 \ {y'^2_{pj}}^t \leq \gamma_p \ {y'^2_{po}}^{t+1} \qquad p \in P,$$

Intermediate

$$\sum_{j=1}^{n} (\lambda_j^1 - \lambda_j^2) {z_{hj}^1}^t \geq (1-\eta_h) {z_{ho}^1}^{t+1} \qquad h \in H_1,$$

$$\sum_{j=1}^{n} \lambda_j^1 = 1$$

$$\sum_{j=1}^{n} \lambda_j^2 = 1$$

$$\lambda_j^1 \geq 0 \ , \ \lambda_j^2 \geq 0 \quad j=1,\ldots,n,$$

$$\theta_i \leq 1 \qquad i \in I_1 \cup I_2,$$

$$\eta_h \leq 1 \qquad h \in H_1,$$

$$\gamma_p \leq 1 \qquad p \in P,$$

$$\varphi_r \geq 1 \qquad r \in R_1, \qquad (8)$$

$$\psi_h \geq 1 \qquad h \in H_2,$$

$$\rho_r \geq 1 \qquad r \in R_2,$$

where $|R_1| = s_1$, $|I_1| = m_1$, $|H_1| = K_1$, $|H_2| = K_2$, $|P| = s_3$, $|R_2| = s_2$ and $|I_2| = m_2$.

Similar to the case of model (5), in regard to the priority of reductions or increases based on the manager's preference, we can consider the parameters α_1, α_2, α_3 and α_4 for the inputs and β_1, β_2 and β_3 for the outputs.

In Model (8), v_{1i} ,v_{2i} , v_{1h} , v_{2p} , u_{1r}, u_{2r} and u_{2h} weight priorities of the inputs and outputs. The afore-mentioned weights are considered in the model based on the decision maker's view.

In addition, first and second stage inputs are classified into two categories of I1 and I2, respectively, therefore two-stage supply chain inputs are equal to I1 ∪ I2, and the aim is to reduce inputs into two separate categories based on priority of the manager.

We also consider inputs first stage of the supply chain as Year of exploitation, Year of discovery, Distance to the city center, Area and Oil in place, which are elements of I1 set. Also, the inputs of the second stage include Salary & benefits, Employee count Shareholder salaries, Current liabilities, Noncurrent liabilities, Year of exploitation and Year of establishment are elements of I2 set in the second stage of the supply chain.

Generally in the proposed models, the index set of outputs includes $R_1 \cup R_2$, but for z_h^1 outputs in the first stage of the network supply chain and also z_h^2outputs in the second stage the supply chain, the index set of h is equal to $H_1 \cup H_2$ which means that H_1 is just index of outputs in the first stage of the network and H_2 is the outputs of the second stage.

To convert fractional model (8) into a linear programming problem form

$$\frac{1}{s_1{+}K_2 + s_2}(\beta_1 \sum_{r\in R_1} u_{1r}\varphi_r + \beta_2 \sum_{r\in R_2} u_{2r}\rho_r + \beta_3 \sum_{h\in H_2} u_{2hh}) = \frac{1}{w}$$

is put and with changing the appropriate variable the model is transformed into a linear form. If we are to obtain the proposed models based on the Slack-based measure, we can obtain the proposed models in form of SBM by modifying the appropriate variables [30].

In model (8), the Shannon entropy measure [31] is used to prioritize the inputs, outputs and intermediate products. Based on the Shannon entropy technique, data are first normalized, and then prioritized accordingly. Therefore, vectors v_{1i}, v_{2i}, v_{2p} and v_{1h} are input weights of the entropy model, and u_{1r}, u_{2r} and u_{2h} are the output weights.

In the enhanced Russell model for two-stage supply chains, the objective is to reduce the inputs and increase the outputs. However, since the model is non-radial, it is important to prioritize these reductions and increases. Therefore, using the Shannon entropy measure [31], the weights of input and output vectors are specified and applied.

$$RLT^{o*} = M^t \left(x_o^{1\ \ t+1}, y_o^{1\ \ t+1}, z_o^{1\ \ t+1}, x_o^{2\ \ t+1}, y_o^{2\ \ t+1}, z_o^{2\ \ t+1},\ y'^{2t+1}_o\right) \quad (8)$$

The following shows the overall form of the MPI in a two-stage supply chain based on the Russell model:

$$\left(\frac{M^t\left(x_o^{1\ t+1}, y_o^{1\ t+1}, z_o^{1\ t+1}, x_o^{2\ t+1}, y_o^{2\ t+1}, z_o^{2\ t+1}, y_o'^{2\ t+1}\right).M^{t+1}\left(x_o^{1\ t+1}, y_o^{1\ t+1}, z_o^{1\ t+1}, x_o^{2\ t+1}, y_o^{2\ t+1}, z_o^{2\ t+1}, y_o'^{2\ t+1}\right)}{M^t\left(x_o^{1\ t}, y_o^{1\ t}, z_o^{1\ t}, x_o^{2\ t}, y_o^{2\ t}, z_o^{2\ t}, y_o'^{2\ t}\right).M^{t+1}\left(x_o^{1\ t}, y_o^{1\ t}, z_o^{1\ t}, x_o^{2\ t}, y_o^{2\ t}, z_o^{2\ t}, y_o'^{2\ t}\right)}\right)^{\frac{1}{2}} \quad (9)$$

The solution of equation (10) is denoted by MQT. In equations (4), (7) and (10), if MPI values were greater than one, DMU_o has progressed from t to t+1, no changes had occurred if the values were equal to one, and if they were less than one, the DMU has regressed from the time of t to t+1.Our article's proposed flowchart is as follows:

Figure 3. Suggested flowchart for calculation of the Malmquist Productivity Index (MPI).

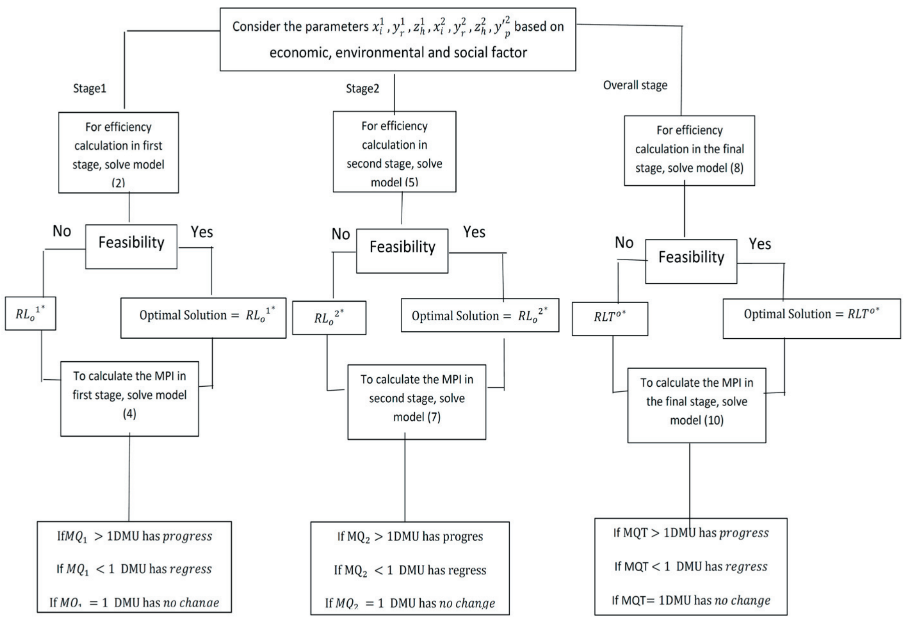

Figure 3. Sustainability Factor

4. Case Study

In this section, we evaluate two-stage supply chains in a number of Iranian oil refineries in two consecutive periods using the MPI.

Oil is not that useful in its raw form, and does not have many applications when extracted from the earth. An oil refinery is an industrial unit, in which crude oil is transformed into more useful forms, such as liquid gas, kerosene, gasoline, diesel fuel, fuel oil, asphalt, tar and other oil products. Oil refineries are usually huge and complex industrial units, in which various units are interconnected by numerous pipeline routes. When oil is gathered in a sedimentary layer, that region is called an oil pool. The sum of multiple oil pools make up an oil field. Oil fields are detected by first choosing a given region, and then studying rock samples and taking seismographs in that region.

Overall, this section considers 9 oil refineries that are active in the form of two-stage supply chains with the inputs and outputs specified in Figure 4. The refineries and oil fields under study are represented by the following abbreviations:

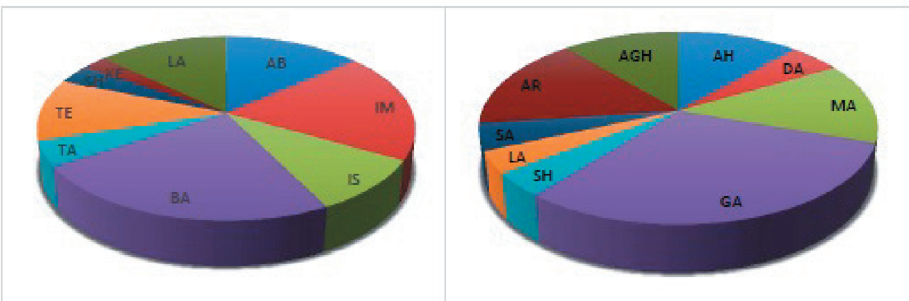

Figure 4. Based on area of Iranian oil fields and refineries under study.

The following are the oil fields under study as presented in Figure 4:

Ahvaz (AH), Darkhoveyn (DA), Marun (MA), Gachsaran (GA), Shadegan (SH), Balal-Lavan (LA), Salman (SA), Aghajari (AGH) and Arvand (AR).

Table 1: Inputs and outputs in two periods.

Environmental factor	Input vector of first stage	xm_i^1	x_i^1=(xm_i^1, xg_i^1)
Economic factor	Input vector of first stage	xg_i^1	x_i^1=(xm_i^1, xg_i^1)
Environmental factor	Output vector of first stage	ym_r^1	y_r^1=ym_r^1
Environmental factor	Output vector of first stage & input vector of second stage	zm_h^1	z_h^1=zm_h^1
Economic factor	Input vector of second stage	xg_i^2	x_i^2= (xg_i^2, xm_i^2)
Environmental factor	Input vector of second stage	xm_i^2	x_i^2= (xg_i^2, xm_i^2)
Environmental factor	Output vector of second stage	ym_r^2	y_r^2=(ym_r^2, yg_r^2, ye_r^2)
Economic factor	Output vector of second stage	yg_r^2	y_r^2=(ym_r^2, yg_r^2, ye_r^2)
Social factor	Output vector of second stage	ye_r^2	y_r^2=(ym_r^2, yg_r^2, ye_r^2)
Economic factor	Output vector of second stage	zg_h^2	z_h^2=zg_h^2
Economic factor	Undesirable output vector of second stage	$y'g_p^2$	y'^2_p=$y'g_p^2$

The 9 oil refineries under study were as follows:

Abadan (AB), Imam Khomeini Shazand (IM), Isfahan (IS), Bandar Abbas (BA), Tabriz (TA), Tehran (TE), Shiraz (SH), Kermanshah (KE) and Lavan (LA).

The inputs and outputs are stratified in table 1 based on economic, environmental and social factors:

Figure 5 provides the descriptions of input and output data in first and second stages: Since components of supply chain are supplier, producer, distributor and consumer, respectively, the supplier and producer are considered in the applied study of two-stage supply chain. In the proposed model, y_r^2 and z_h^2 can be the same. In separating outputs of the second stage, which are essentially the outputs of the producer, if the outputs are considered as inputs of distributors, they are simply presented by z_h^2 parameters such as Average Product and Cost of Goods Sold. If the Greanspace of refineries is not considered as an input for the distributor, then in the case study, the optimal output is y_r^2 and the output of the second stage is z_h^2, in which using the variables ρ_r and p$_h$to increase them is considered.

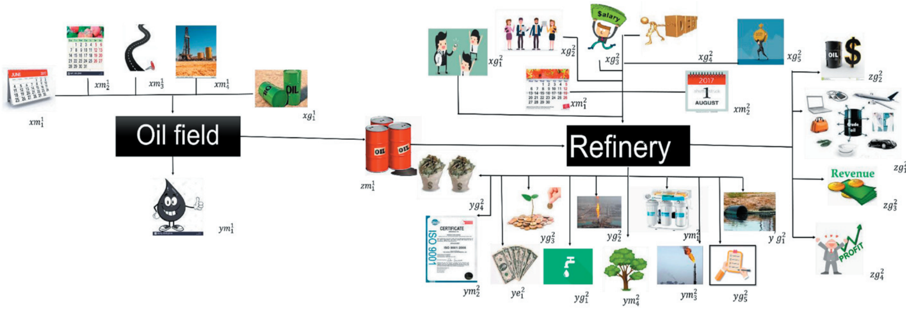

Figure 5. Definition of input, output and intermediate vectors in two consecutive periods.

The reasons for not using stages related to the distributor and consumer in the case study are as follows:

(A) The distribution of oil produced by Iranian oil refineries is carried out nationwide and under the auspices of the Ministry of Oil.

(B) Refinery consumers are determined by the sales process at the Ministry of Oil.

The first portion of the data is related to oil fields, and oil refineries are considered in the second stage. Tables 2 and 3 present the input and output data of the oil fields and refineries in the first period. Input and output data related to the second period are provided in Tables 4 and 5.

Table 2: Inputs and outputs of the oil fields in the first period.

DMU	xm_i^1				zm_h^1	ym_r^1	xg_i^1
1	402	20	1337	1338	37000	750000	70241
2	173	45	1343	1343	1300	160000	5500
3	469	40	1342	1345	22	520000	48250
4	1050	220	1306	1308	23700	480000	70141
5	138	60	1347	1367	66000	66000	4189
6	125	94	1378	1381	117000	40000	389000
7	154	144	1343	1346	1600	60000	5171
8	388.5	90	1315	1317	174	300000	31245
9	546	50	1387	1387	150	20000	1129

Table 3-a: Inputs and outputs of the oil refineries in the first period.

DMU	xm_i^2	xm_i^2	xg_i^2				
1	1288	1291	2052393	23565172	1452356	3727	139105
2	1368	1372	9763547	16576165	6121981	819	19691
3	1355	1358	10746968	10307055	13395933	1243	42082
4	1371	1376	180023	8049827	15322853	1248	29836
5	1353	1357	192582	3148025	3326676	718	30206
6	1344	1348	410265	16860019	895863	1039	70257
7	1349	1352	176818	1776556	1512249	672	22293
8	1347	1350	133533	1542766	242076	401	6640
9	1354	1355	65044	2460131	850347	335	18902

Table 3-b: Inputs and outputs of the oil refineries in the first period.

DMU	ym_r^2	ym_r^2	ym_r^2	ym_r^2	zg_h^2			
1	126	4123924	3	2918	114578181	9674475	8262228	1035
2	53	5782317	4	976318	45736691	5619719	5043448	7623
3	945	2781415	2	200125	105174553	8781645	6771546	10234
4	276	6980121	6	1112	78797156	5963566	4328457	9765
5	44	1314178	3	955	30804040	2664567	2017632	2976
6	184	3241672	5	2115	63996018	5280908	4066575	4993
7	297	1000000	2	218	15450320	1202313	913703	1445
8	19	739256	4	249	6609781	217706	168104	690
9	48	3921112	5	15	8413772	75521	75006	1802

Table 3-c: Inputs and outputs of the oil refineries in the first period.

DMU	$y'g_p^2$	ye_r^2	yg_r^2				
1	4732	426750	183373	32862	1296702	326	2718
2	16659	61685	567526	259658	5646390	118	1143
3	7438	162335	74016	372414	4162258	293	2117
4	4261	87667	229642	563214	13800000	305	5114
5	5371	85065	129135	54043	1942105	94	876
6	6812	100100	95715	94509	61220	236	1217
7	2517	103546	11865	136309	1027549	44	418
8	892342	50177	9184	20114	134879	19	650
9	1110	37787	12715	77682	1143422	41	21

Table 4: Inputs and outputs of the oil fields in the second period.

DMU	$\boldsymbol{xm_i^1}$				$\boldsymbol{xg_i^1}$	$\boldsymbol{zm_h^1}$	$\boldsymbol{ym_r^1}$
1	402	20	34651	740142	65500	34651	740142
2	173	45	950	154231	5000	950	154231
3	469	40	19	501219	46700	19	501219
4	1050	220	20651	469549	66700	20651	469549
5	138	60	61276	65100	3300	61276	65100
6	125	94	111575	35121	293000	111575	35121
7	154	144	950	50970	4500	950	50970
8	388.5	90	147	19746	30200	147	19746
9	546	50	89	17191	1000	89	17191

Table 5-a: Inputs and outputs of the oil refineries in the second period

DMU	$\boldsymbol{xm_i^2}$	$\boldsymbol{xm_i^2}$	$\boldsymbol{zg_h^2}$				$\boldsymbol{y'g_p^2}$
1	1288	1291	98562638	2040505	1761123	12075	3942
2	1368	1372	39479149	1744811	843754	9150	15104
3	1355	1358	92048590	4957965	3987876	11450	5919
4	1371	1376	850017002	156211	12389	10750	3708
5	1353	1357	26769423	452937	367349	3462	2342
6	1344	1348	59238408	1475957	1212182	7358	5218
7	1349	1352	13632935	369413	293832	1731	1152
8	1347	1350	6399107	818971	781860	737	792431
9	1354	1355	6934817	104646	102586	1864	908

Table 5-b: Inputs and outputs of the oil refineries in the second period.

DMU	$\boldsymbol{ym_r^2}$				$\boldsymbol{yg_r^2}$				
1	188	4950000	5	3942	108125	1601380	1296702	400	3942
2	87	6000000	5	1192320	192034	411986	5646390	250	2934
3	1145	3400000	4	2949	55192	248975	4162258	375	3154
4	358	7000000	8	1475	473708	124014	13800000	320	6036
5	52	2000000	4	1273	65090	109709	1942105	110	1949
6	293	4000000	6	2423	84116	185674	61220	250	2774
7	330	1140000	4	600	18802	164809	1027549	58	1148
8	20	843898	5	276	7295	42307	134879	21000	720
9	57	4000000	6	26	21935	105379	1143422	60	26

Table 5-c: Inputs and outputs of the oil refineries in the second period.

DMU	xg_i^2				
1	4732916	22779935	2509030	3497	163959
2	10470192	20285143	6368159	889	29785
3	485621	14936671	13637778	1240	37764
4	223030	8031656	14048191	1192	41521
5	274145	2981522	2373062	699	90226
6	456641	18841639	486472	1025	194580
7	220427	1834087	1265830	633	24014
8	145814	1257999	539823	419	24048
9	83538	4371837	747761	367	31170

Table 6 shows the efficiency scores produced by the enhanced Russell model, as well as the MPIs, in the two periods under study.

The second column provides the efficiency scores in the first period, the column three shows the scores in a combination of first and second periods, column four includes the scores in a combination of second and first periods, the efficiency scores of the second period are presented in column five, and the sixth column involves the calculated MPIs in the first stage.

Table 6: Efficiency scores of the first stage in different periods.

DMU	$(\mathbf{RL}^1)_{t+1}$	$(\mathbf{RL}^1)_{\mathrm{t+1,t}}$	$(\mathbf{RL}^1)_{\mathrm{t,t+1}}$	$(\mathbf{RL}^1)_t$	MQ_1
1	1	0.985	1	1	1.007
2	1	0.969	0.349	1	0.600
3	0.860	0.847	0.292	0.863	0.587
4	1	0.599	0.206	1	0.586
5	1	0.768	0351.	1	0. 676
6	0.578	0.542	0248.	1	0.514
7	1	0.613	0.284	1	0.680
8	1	0.534	0.251	1	0.685
9	1	0.781	1.197	1	1.238

Table 7 Presents the MPIs and the efficiency scores of the second stage in different periods.

Similar to Table 6, the second column of Table 7 shows the efficiency score in first periods, the third column shows efficiency scores in the first and second periods, the fourth column presents efficiency scores in the second and first periods, the fifth column shows the second MPI in the second stage.

Table 7: Efficiency scores of the second stage in different periods.

DMU	$(RL^2)_{t+1}$	$(RL^2)_{t+1,t}$	$(RL^2)_{t,t+1}$	$(RL^2)_t$	MQ_2
1	0.514	1	0.448	1	0.933
2	0.584	1	0.630	1	1.038
3	0.450	1	0.510	1	1.063
4	0. 608	1	0.707	1	1.078
5	0.726	1	0.635	1	0.935
6	0.603	1	0.669	1	1.053
7	0. 949	1	0.865	1	0.954
8	1.461	1	1	1	0.204
9	1.187	1	0.903	1	0.871

In Table 8 efficiency scores of the combination of both stages are calculated in different periods, and overall MPIs are provided.

Table 8: Overall efficiency scores in different periods.

DMU	$\mathbf{(RLT)_{t+1}}$	$\mathbf{(RLT)_{t+1,t}}$	$\mathbf{(RLT)_{t,t+1}}$	$\mathbf{(RLT)_t}$	**MQT**
1	0.995	0.645	0.608	1	0.968
2	0.993	0.604	0.661	1	1.042
3	0.997	0.588	0.533	0.879	0.950
4	0.996	0.568	0.542	1	0.974
5	0.984	0.653	0.705	1	1.030
6	0.778	0.542	0.615	1	0.939
7	0.990	0.736	0.759	1	1.010
8	0.997	1.034	1	1	0.981
9	0.991	0.958	1.733	1	1.338

Table 9 shows the levels of progress and regression in different periods using the MPI.

Table 9: Level of progress and regression in the first, second and overall stages.

DMU	First Stage	Second Stage	Overall Stage
1	+	-	-
2	-	+	+
3	-	+	-
4	-	+	-
5	-	-	+
6	-	+	-
7	-	-	+
8	-	-	-
9	+	-	+

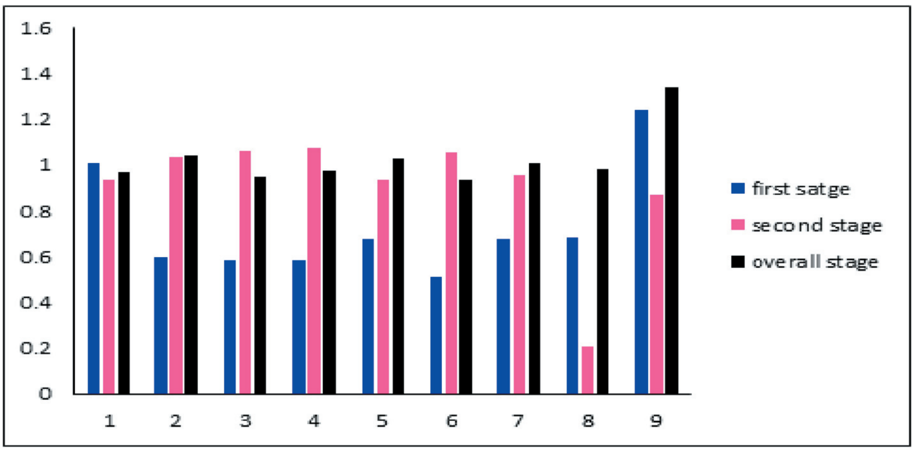

Figure 6. MPI in first stage, second stage and overall stage.

As can be observed in Table 9 and Figure 6, DMU_1 and DMU_9 had progressed in the first stage, but the other units had regressed. In second stage, DMU_2, DMU_3, DMU_4 and DMU_6 had regressed, and the other DMUs had progressed. Finally, after considering both stages of the supply chain, we found that DMU_2, DMU_5, DMU_7 and DMU_9 had overall progressed.

The main problem and challenge was in DMU_5 and DMU_7, because they both had regressed in stages one and two, but both had progressed in the final stage with MPIs of 1.030 and 1.010, respectively. Although, this can be explained by the following arguments:

A) Since all constraints corresponding to input, output and intermediate vectors are applied in the final stage, evaluation is done more thoroughly. In this regard, even though DMUs 5 and 7 had regressed in each individual stage, they were overall in a relatively stable or improved condition.

B) The MPIs calculated for DMU_5 and DMU_7 indicate progress in two consecutive periods. however, this finding was not significant.

Overall, DMU_2 and DMU_9 had a good progress with MPIs of 1.042 and 1.338, respectively. These two units had substantially progressed in both stages one and two. Thus, our results suggest that we cannot draw any conclusions regarding progress and regression merely based on MPIs in stages one and two.

The following describes the main reasons and motivations of this research for evaluating the oil refineries in Iran with the use of the Malmquist productivity index in a two-stage network:

1. The raw materials used in oil refineries are supplied from oil wells, and the final products include liquid gas, gasoline, kerosene, paraffin, asphalt, and bitumen.
2. In order to find suitable benchmarks for inefficient units, the non-radial Enhanced Russel Measure (ERM) is used, which divides the mean reduction in inputs by the mean increase in outputs with the objective to reduce all inputs and increase all outputs. Therefore, the MPI is calculated in two different time periods through this method.

The most significant issue in MPI calculation in two-stage networks relates to the calculation of overall MPI, where the restrictions of network stages (1) and (2) are combined. Moreover, the objective function of the new model is based on the enhanced Russel measure.

Remark 1. Three strategies are proposed for observing relationship $3\,(m+s) \leqslant n$:

1) Delphi method[32]: in this technique, after compiling the views of all experts and the merging inputs and outputs, the number of inputs and outputs are reduced in order to obtain $3\,(m+s) \leqslant n$ relation.

2) AHP method: In AHP method, the criteria and sub-criteria are specified and a pairwise comparison matrix is developed, and then weights of criteria and sub-criteria are obtained [33].

As there are seventeen outputs in the case study, we specify the criteria and sub-criteria for them, then by using the analytic hierarchy process (AHP), weights of outputs are determined and the outputs with weight of less than 30 % are eliminated in order to obtain $3\,(m+s) \leqslant n$ relation.

3) Using cause and effect relationship methods, we accurately analyze inputs and outputs of the supply chain. Then, by using statistical methods and calculating the correlation between them, we consider just inputs and outputs with the most impact.

Nevertheless, since data of supply chains in the present paper is real and there is no linear relationship between input (or output) categories, the above strategies may not provide accurate performance evaluation of the units. Therefore, we conducted a performance evaluation by considering the weight priority in the inputs and outputs.

In DEA, the relationship between the number of inputs (m) and outputs (s) and DMUs (n), 3 (m + s) $\leqslant$n is of great importance. But if 3 (m + s) $\leqslant$n does not hold, all decision-making units may be efficient, meanwhile it can be controlled using weight constraints, but this may have disadvantages. In the present article, due to the using real data of the supply chains of 9 oil refineries, two ideas have been utilized in determining inputs and outputs:

(A) Considering input, output and intermediate parameters separately (according to Section 4 of the article)

(B) scale the inputs and outputs and integrate them in a way that relation holds

Therefore, for the present paper authors have considered Idea A since for calculating the Malmquist Productivity Index with the obtained efficiency value, progress and regression of DMUs are determined.

Remark 2. The reason for using the proposed models under variable- returns to scale technology is based on the case study. In studying the supply chain of 9 oil refineries in two consecutive periods in Iran, the input of oil in place, for instance, doesn't enjoy the Ray Unboundedness condition, since with the increase in the value of$\lambda > 0$, it is not possible to the produce it. Also, if we consider the number of employees in the second stage of the supply chain as inputs, we can observe that Ray Unboundedness condition, i.e. the increase of positive arbitrage coefficient, in inputs and outputs is not met. If we consider the capacity in the second stage, which is assumed to be yg_2^2, it is still not possible to increase the capacity with any positive multiplier.

Therefore if (x,y,z)∈ T, then(x,y,z)∈T does not hold. Thus, based on the case study, variable- returns to scale technology is considered.

5. Conclusion

Evaluation of the supply chain process is one of utmost importance in all organizations, as economic, environmental and social factors greatly influence the supply chain. In this regard, tools and techniques such as DEA provide the opportunity to identify the strengths and weaknesses In our evaluation of two-stage networks in 9 Iranian oil refineries in the years 2006-2007, the Malmquist productivity index (MPI) specified the amount of progress or regression in each stage. However, the main result pertained to the overall mode, where the restrictions of first stage and second stage were combined. Therefore, companies with regression in both stages had obviously regressed in the overall mode, and vice versa. Although, in the case of regression in one stage and progress in the other, overall efficiency and the MPI would determine the level of progress or regression in every DMU. Evaluating the units based on the enhanced Russell measure and then using the Malmquist productivity index would make it possible to specify the exact level of inefficiency for every input and output in each separate stage. In the present study, this possibility to evaluate the progress and regression of each oil refinery has been specified in detail based on the proposed models.

The proposed algorithm for calculating the MPI in two-stage supply chains is convergent, as the ERM is used in the first stage for efficiency calculation at the times of t and t+1, which always has a feasible and definable solution due to the presence of constraints $\theta_i \leqslant 1$ and $\varphi_r \geqslant 1$. in addition, $0 < {RL_o}^{1^*} \leqslant 1$.

Thereby, model (4) is used to calculate the MPI in the first stage of the supply chain. Obviously, at the beginning of the algorithm, the types of input, output and intermediate parameters are distinguished based on environmental, social and economic factors.

Similarly, in the second stage of the supply chain, we will arrive at $0 < {RL_o}^{2^*} \leqslant 1$ using model (5), and $|H_2|$, $|P|$, $|R_2|$ and $|I_2|$ are finite. On the other hand, since there are a finite number of DMUs in the supply chain, the efficiency value is then calculated in a finite number of linear programming solutions and entered into model (7).

The proposed models that have a fractional form are converted into a linear form by using the transformation technique developed by Charnes and Cooper (Chames et al., 1978)

In order to calculate the overall efficiency scores, model (8) is developed by combining the constraints in stages one and two, where $0 < RLT^{o*} \leqslant 1$. Moreover, since the ERM is used at the times of t and t+1, it is made possible for the algorithm to be convergent in a finite number of iterations.

We also conducted an applied study of Iranian oil wells and refineries formed as sustainable supply chains in two consecutive periods, with consideration to the conditions of sustainability. As mentioned in the previous section, 22% of the decision-making units (DMU_1 & DMU_9) had progressed in the first stage of the supply chain, and 78% had regressed (DMU_2, DMU_3, DMU_4, DMU_5, DMU_6, DMU_7 & DMU_8). In the second stage, 44% of the units had progressed (DMU_2, DMU_3, DMU_4 & DMU_6) and 56% regressed (DMU_1, DMU_5, DMU_7, DMU_8 & DMU_9). Similarly, we observe progress in 44% of the units (DMU_2, DMU_5, DMU_7 & DMU_9) and regression in 56% (DMU_1, DMU_3, DMU_4, DMU_6 & DMU_8) in the final stage. Generally, the progress or regression witnessed in the first stage is not a conclusive determinant for the final stage. For instance, units 2-8 had all regressed in the first stage, while it was found that DMUs 2, 5 and 7 had overall progressed in the final stage. In addition, any progress in the second stage alone will not affect the final stage either.

For example, DMU_3, DMU_4 and DMU_6 had progressed in second stage, but showed an overall regression in the final stage. For future research, we suggest using the assumption of returns to scale in the supply chain, and finding suitable benchmarks in a sustainable supply chain based on the MPI.

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