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An Application of Function Sequences to the Darbo's Theorem with Integral Type Transformations

V. Karakaya*
Yildiz Technical University

D. Sekman
Ahi Evran University

Abstract. The purpose of this work is to investigate the behavior of function sequences under integral type transformations as a generalization of Darbo's theorem. It is also to obtain the existence of the fixed point of this transformation involving the function sequences. In addition, the study will be explained with an interesting example.

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1. Introduction

Fixed point theory is a fascinating theory that has been developed to demonstrate the existence and uniqueness of the solution of a wide range of problems which arise in nonlinear analysis, applied mathematics, and many related disciplines such as game theory, economics, medicine, biology, and physics.

The three important theorems that stand out in fixed point theory are the Schauder [12], Brouwer [7], and Darbo [9] theorems. Among these, Schauder's theorem reveals an important generalization of Brouwer's theorem from finite-dimensional spaces to infinite-dimensional spaces. However, Darbo's theorem

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*Corresponding author

provides an effective method for the existence and uniqueness of fixed points of the set-valued transformations of non-compact operators. Recently, many researches have been devoted to the solution of the classes of integral equations given in [1, 6, 13] using the measure of noncompactness and Darbo's fixed point theorem.

On the other hand, contraction operators whose origin date back to Banach's contraction principle [3] and their generalizations have a central role in obtaining the solution of many important problems arise in various disciplines of sciences and engineering and thus, have attracted the attention of numerous researchers. As a generalization of the Banach contraction principle, Berzig [5] introduced the concept of shifting distance function. Using this concept, Samadi and Ghaemi [11] presented some generalizations of Darbo's theorem with an application to mixed-type integral equations. Later, Cai and Liang [8] obtained some new generalizations of the Darbo's fixed point theorem using integral-type transformations. Quite recently, Karakaya et al. [10] introduced a new concept of function sequences with shifting distance function and proved some new Darbo type theorems. In this work, we investigate the behavior of function sequences under integral type transformations as a generalization of Darbo's theorem. It is also to obtain the existence of the fixed point of this transformation under the function sequences. In addition, the study will be explained with an interesting example.

2. Preliminaries

Let C be a nonempty subset of a Banach space X. We define $\overline{C}$ and $Conv(C)$ the closure and closed convex hull of C, respectively. Also, we denote by M_X which is the family of all nonempty bounded subsets of X and N_X that is subfamily consisting of all relatively compact subsets of X. Throughout this work, we will show uniform convergence according to n in function sequences with the symbol " $\rightrightarrows$ ". Also we denote $\mathbb{N}$, $\mathbb{R}$, $\mathbb{R}^+$ which are natural number, real number and positive real number, respectively.

Definition 2.1. [see; [4]] *A mapping $\mu : M_X \to \mathbb{R}^+$ is called a measure of noncompactness if it satisfies the following conditions:*

(M_1) The family Ker $\mu = \{E \in M_X : \mu(E) = 0\}$ is nonempty and Ker $\mu \subseteq N_X$,

(M_2) $E \subseteq F \Rightarrow \mu(E) \leqslant \mu(F)$,

(M_3) $\mu(\overline{E}) = \mu(E)$, where $\overline{E}$ denotes the closure of E,

(M_4) $\mu(Conv\ (E)) = \mu(E)$,

(M_5) $\mu(\lambda E + (1-\lambda)F) \leqslant \lambda\mu(E) + (1-\lambda)\mu(F)$ for $\lambda \in [0,1]$,

(M_6) If $\{E_n\}$ is a sequence of closed sets in M_X such that $E_{n+1} \subseteq E_n$ and $\lim_{n\to\infty} \mu(E_n) = 0$, then the following intersection is nonempty

$$E_\infty = \bigcap_{n=1}^{\infty} E_n,$$

for $n = 1, 2, \cdots$. If (M_4) holds, then $E_\infty \in \text{Ker } \mu$. Let $\lim_{n\to\infty} \mu(E_n) = 0$. As $E_\infty \subseteq E_n$ for each $n = 0, 1, 2, ...$; by the monotonicity of μ, we obtain

$$\mu(E_\infty) \leqslant \lim_{n\to\infty} \mu(E_n) = 0.$$

So, by (M_1), we get that E_∞ is nonempty and $E_\infty \in \text{Ker } \mu$.

Theorem 2.2. [see; [12]] *Let C be a closed and convex subset of a Banach space X. Then every compact, continuous map $T : C \to C$ has at least one fixed point.*

Theorem 2.3. [see; [9]] *Let C be a nonempty, bounded, closed and convex subset of a Banach space X and let $T : C \to C$ be a continuous mapping. Suppose that there exists a constant $k \in [0, 1)$ such that*

$$\mu(TE) \leqslant k\mu(E)$$

for any subset E of C, then T has a fixed point.

Definition 2.4. [see; [5]] *Let $\psi, \phi : [0, \infty) \to \mathbb{R}$ be two functions. The pair (ψ, ϕ) is said to be shifting distance function, if the following conditions hold:*

(*i*) for $u, v \in [0, \infty)$, if $\psi(u) \leqslant \phi(v)$, then $u \leqslant v$,

(*ii*) for $\{u_k\}, \{v_k\} \subset [0, \infty)$ with $\lim_{k\to\infty} u_k = \lim_{k\to\infty} v_k = w$, if $\psi(u_k) \leqslant \phi(v_k)$ *for all $k \in \mathbb{N}$, then $w = 0$.*

Theorem 2.5. [see; [11]] *Let C be a nonempty, bounded, closed and convex subset of the Banach space X. Suppose that $T : C \to C$ is a continuous mapping such that*

$$\psi(\mu(TE) \leqslant \phi(\mu(E)) \tag{1}$$

for any nonempty $E \subset C$, where μ is an arbitrary measure of noncompactness. Then, T has a fixed point in C.

Theorem 2.6. [see; [8]] *Let C be a nonempty, bounded, closed and convex subset of the Banach space X. Suppose that $T : C \to C$ is a continuous mapping such that*

$$\psi\left(\int_0^{\mu(TE)} \varphi(t)dt\right) \leqslant \phi\left(\int_0^{\mu(E)} \varphi(t)dt\right) \tag{2}$$

for any nonempty $E \subset C$, where μ is an arbitrary measure of noncompactness and $\psi, \phi : [0, \infty) \to \mathbb{R}$ are a pair of shifting distance functions. Moreover, let $\varphi : [0, \infty) \to [0, \infty]$ be a Lebesgue-integrable function, which is summable on each compact subset of $[0, \infty)$ and $\int_0^\varepsilon \varphi(t)dt > 0$ for each $\varepsilon > 0$. Then, T has at least one fixed point in C.

Definition 2.7. [see; [10]] *Let $\psi_n, \phi_n : [0, \infty) \to \mathbb{R}$ be two function sequences. Let the pair (ψ, ϕ) be shifting distance function. The pair (ψ_n, ϕ_n) is said to be function sequences with shifting distance function property which satisfy the following conditions:*

(i) for $u, v \in [0, \infty)$, if $\psi_n(u) \rightrightarrows \psi(u), \phi_n(v) \rightrightarrows \phi(v)$ and $\psi(u) \leqslant \phi(v)$, then $u \leqslant v$,

(ii) for $\{u_k\}, \{v_k\} \subset [0, \infty)$ with $\lim\limits_{k\to\infty} u_k = \lim\limits_{k\to\infty} v_k = w$, if $\psi_n(u_k) \rightrightarrows \psi(u_k), \phi_n(v_k) \rightrightarrows \phi(v_k)$ and $\psi(u_k) \leqslant \phi(v_k)$ for all $k \in \mathbb{N}$, then $w = 0$.

Lemma 2.8. [see; [10]] *Let $\psi_n, \phi_n : [0, \infty) \to R$ be two function sequences . Assume that the function sequences hold following conditions:*

(i) if $\{\psi_n\}$ upper semi-continuous function sequences and $\psi_n \leqslant \psi_{n+1}$, then $\psi_n \to \psi$ is uniform convergence according to n,

(ii) if $\{\phi_n\}$ lower semi-continuous function sequences and $\phi_n \geqslant \phi_{n+1}$, then $\phi_n \to \phi$ is uniform convergence according to n.

Then (ψ_n, ϕ_n) is the function sequence with shifting distance function property.

Theorem 2.9. [see; [10]] *Let C be a nonempty, bounded, closed and convex subset of the Banach space X. Suppose that $T : C \to C$ is a continuous mapping such that*

$$\psi_n(\mu(TE)) \leqslant \phi_n(\mu(E)) \tag{3}$$

for any nonempty $E \subset C$, where μ is an arbitrary measure of noncompactness and $\psi_n, \phi_n : [0, \infty) \to \mathbb{R}$ be the function sequences with shifting distance function property. Then, T has a fixed point in C.

3. Main Results

Theorem 3.1. *Let C be a nonempty, bounded, closed and convex subset of the Banach space X. Suppose that $T : C \to C$ is a continuous mapping such that*

$$\psi_n\left(\int_0^{\mu(TE)} \varphi(t)dt\right) \leqslant \phi_n\left(\int_0^{\mu(E)} \varphi(t)dt\right) \tag{4}$$

for any nonempty $E \subset C$, where μ is an arbitrary measure of noncompactness and $\psi_n, \phi_n : [0, \infty) \to \mathbb{R}$ be the function sequences with shifting distance *function property. Furthermore, let* $\varphi : [0, \infty) \to [0, \infty]$ *be a Lebesgue-integrable function, which is summable on each compact subset of* $[0, \infty)$ *and* $\int_0^{\varepsilon} \varphi(t)dt > 0$ *for each* $\varepsilon > 0$*. Then,* T *has a fixed point in* C*.*

Proof. We define a sequence $\{E_k\}$ such that $E_0 = E$ and $E_k = Conv\,(TE_{k-1})$ for all $k \geqslant 1$. Then we get

$$\begin{aligned} TE_0 &= TE \subseteq E = E_0 \\ E_1 &= Conv\,(TE_0) \subseteq E = E_0. \end{aligned}$$

If this process is continued, we have

$$E_0 \supseteq E_1 \supseteq E_2 \supseteq \cdots \supseteq E_k \supseteq \cdots .$$

If there exists an integer $k \geqslant 0$ such that $\mu\,(E_k) = 0$, then E_k is relatively compact and since

$$TE_k \subseteq Conv\,(TE_k) = E_{k+1} \subseteq E_k,$$

Theorem 2.2 implies that T has a fixed point on the set E_k for all $k \geqslant 0$. Now, we assume that $\mu(E_k) > 0$ for all $k \geqslant 0$. By using (4) we have

$$\begin{aligned} \psi_n \left(\int_0^{\mu(E_{k+1})} \varphi(t)dt \right) &= \psi_n \left(\int_0^{\mu(Conv(TE_k))} \varphi(t)dt \right) \qquad (5) \\ &= \psi_n \left(\int_0^{\mu(TE_k)} \varphi(t)dt \right) \\ &\leqslant \phi_n \left(\int_0^{\mu(E_k)} \varphi(t)dt \right). \end{aligned}$$

Suppose that (4) holds. Then we get that $\left\{ \int_0^{\mu(E_k)} \varphi(t)dt \right\}$ is a decreasing sequence of positive real numbers by (ii) of Definition 2.7 and there exists $r \geqslant 0$ such that $\int_0^{\mu(E_k)} \varphi(t)dt \to r$ as $k \to \infty$. By using together with (5) and Lemma 2.8, we get $\psi_n \left(\int_0^{\mu(E_{k+1})} \varphi(t)dt \right) \rightrightarrows \psi \left(\int_0^{\mu(E_{k+1})} \varphi(t)dt \right)$ and

$$\psi \left(\int_0^{\mu(E_{k+1})} \varphi(t)dt \right) = \psi \left(\int_0^{\mu(TE_k)} \varphi(t)dt \right). \qquad (6)$$

Also, if $\int_0^{\mu(E_k)} \varphi(t)dt \to r$ as $k \to \infty$, then $\int_0^{\mu(E_{k+1})} \varphi(t)dt \to r$ as $k \to \infty$. Hence, from Theorem 2.6, we have

$$\lim_{k\to\infty} \psi \left(\int_0^{\mu(E_{k+1})} \varphi(t)dt \right) = \lim_{k\to\infty} \psi \left(\int_0^{\mu(TE_k)} \varphi(t)dt \right) \leqslant \lim_{k\to\infty} \phi \left(\int_0^{\mu(E_k)} \varphi(t)dt \right).$$

Indeed, $\psi(r) \leqslant \phi(r)$. From (ii) of Definition 2.7, we obtain $r = 0$. Hence, we have $\int_0^{\mu(E_k)} \varphi(t)dt \to 0$ as $k \to \infty$. On the other hand, since $E_{k+1} \subseteq E_k$, we obtain that $TE_k \subseteq E_k$ and $\int_0^{\mu(E_k)} \varphi(t)dt \to 0$ as $k \to \infty$. Using (M_6) of Definition 2.1, $E_\infty = \bigcap\limits_{k=1}^{\infty} E_k$ is nonempty, closed, convex, and invariant under T. Hence the mapping T belong to Ker μ. Therefore, Schauder's fixed point theorem implies that T has a fixed point in $E_\infty \subset E$. $\square$

Example 3.2. We denote the following function sequences by

$$\psi_n(u) = \frac{4n(1+u)+2u+1}{2n+1}, \phi_n(v) = \frac{n^2(2+v)+1}{n^2}.$$

It holds the conditions of Definition 2.7. We assume that

$$u = \int_0^{\mu(TE)} \varphi(t)dt, \; v = \int_0^{\mu(E)} \varphi(t)dt,$$

we get

$$\int_0^{\mu(TE)} \varphi(t)dt \leqslant \frac{1}{2} \int_0^{\mu(E)} \varphi(t)dt.$$

If we take $\varphi(t) = 1$, according to Darbo's fixed point theorem, T has a fixed point.

Remark 3.3. *If we take $\varphi(t) = 1$ for $t \in [0, \infty)$ in Theorem 3.1 such that*

$$\psi_n \left(\int_0^{\mu(TE)} \varphi(t)dt \right) = \psi_n(\mu(TE)) \leqslant \phi_n(\mu(E)) = \phi_n \left(\int_0^{\mu(E)} \varphi(t)dt \right),$$

then we obtain Theorem 3.4 given in [10].

Remark 3.4. *Taking $\varphi(t) = 1$, $\psi_n(t) = I_n(t)$ and $\phi_n(t) = kI_n(t)$ such that $I_n \rightrightarrows I$ in Theorem 3.1, then we have*

$$\mu(TE) = \psi_n \left(\int_0^{\mu(TE)} \varphi(t)dt \right) \leqslant \phi_n \left(\int_0^{\mu(E)} \varphi(t)dt \right) = k\mu(E),$$

so we get Darbo's fixed point theorem, where $k \in [0, 1)$.

If we take $(\psi_n) = (I_n)$ such that $\lim\limits_{n\to\infty} I_n = I$ uniformly convergence for all $n \in \mathbb{N}$ in Theorem 3.1, we obtain the following result.

Corollary 3.5. *Let C be a nonempty, bounded, closed and convex subset of the Banach space X. Suppose that $T : C \to C$ is a continuous function such that*

$$I_n \left(\int_0^{\mu(TE)} \varphi(t)dt \right) \leqslant \phi_n \left(\int_0^{\mu(E)} \varphi(t)dt \right)$$

for any nonempty subset of $E \subset C$*, where* μ *is an arbitrary measure of noncompactness and* $\phi_n : [0,\infty) \to \mathbb{R}$ *be a function sequences such that*

(a) for $u, v \in [0,\infty)$, if $I_n(u) \leqslant \phi_n(v)$, then $u \leqslant v$,

(b) for $\{u_k\}, \{v_k\} \subset [0,\infty)$ with $\lim_{k\to\infty} u_k = \lim_{k\to\infty} v_k = w$, if $I_n(u_k) \leqslant \phi_n(v_k)$ for all $n, k \in \mathbb{N}$, then $w = 0$.

Also let $\varphi : [0,\infty) \to [0,\infty)$ be a Lebesgue-integrable function, which is summable on each compact subset of $[0,\infty)$ and $\int_0^\varepsilon \varphi(t)dt > 0$ for each $\varepsilon > 0$. Then, T has a fixed point in C.

Theorem 3.6. *Let* C *be a nonempty, bounded, closed and convex subset of the Banach space* X*. Suppose that* $T : C \to C$ *is a continuous mapping such that*

$$\psi_n\left(\int_0^{\mu(TE)} \varphi(t)dt\right) \leqslant \psi_n\left(\int_0^{\mu(E)} \varphi(t)dt\right) - \phi_n\left(\int_0^{\mu(E)} \varphi(t)dt\right) \qquad (7)$$

for any nonempty $E \subset C$*, where* μ *is an arbitrary measure of noncompactness and* $\psi_n, \phi_n : [0,\infty) \to \mathbb{R}^+$ *be a pair with shifting distance function property. Also the pair* (ψ,ϕ) *is two nondecreasing and continuous functions satisfying* $\psi(t) = \phi(t)$ *if and only if* $t = 0$*. Furthermore, let* $\varphi : [0,\infty) \to [0,\infty)$ *be a Lebesgue-integrable function, which is summable on each compact subset of* $[0,\infty)$ *and* $\int_0^\varepsilon \varphi(t)dt > 0$ *for each* $\varepsilon > 0$*. Then,* T *has a fixed point in* C*.*

Proof. Suppose that (7) holds. If by taking limit on (7), we have

$$\psi\left(\int_0^{\mu(TE)} \varphi(t)dt\right) \leqslant \psi\left(\int_0^{\mu(E)} \varphi(t)dt\right) - \phi\left(\int_0^{\mu(E)} \varphi(t)dt\right). \qquad (8)$$

Along with that, by using hypothesis in expression, we suppose that

$$\psi\left(\int_0^{\mu(E)} \varphi(t)dt\right) = \phi\left(\int_0^{\mu(E)} \varphi(t)dt\right).$$

Then we get $\int_0^{\mu(E)} \varphi(t)dt = 0$. By using the conditions of Theorem 3.1, E is relatively compact and then Theorem 2.2 implies that T has a fixed point in C. Conversely, we suppose that $\mu(E) = 0$. Then we can show the following form in (8)

$$\psi\left(\int_0^{\mu(E)} \varphi(t)dt\right) = \phi\left(\int_0^{\mu(E)} \varphi(t)dt\right).$$

Since $\int_0^{\mu(E)} \varphi(t)dt = 0$, it is easy to see that E is relatively compact. From the condition of Theorem 2.2, we say that T has a fixed point in C. Also since

$(\psi, \phi) \in \mathbb{R}^+$, $\int_0^{\mu(TE)} \varphi(t)dt = 0$. So by repeating the conditions of Theorem 3.1, we obtain that T belong to Ker μ. As a result, mapping T has a fixed point in $E_\infty \subset E$. $\square$

Theorem 3.7. *Let C be a nonempty, bounded, closed and convex subset of the Banach space X. Suppose that $T : C \to C$ is a continuous mapping such that*

$$\psi_n \left(\int_0^{\mu(TE)} \varphi(t)dt \right) \leqslant \phi_n \left(\int_0^{\mu(E)} \varphi(t)dt \right) - \theta_n \left(\int_0^{\mu(E)} \varphi(t)dt \right) \quad (9)$$

for any nonempty $E \subset C$, where μ is an arbitrary measure of noncompactness and $\psi_n, \phi_n, \theta_n : [0, \infty) \to \mathbb{R}^+$ be function sequences with shifting distance function property, triplet functions $(\psi_n, \phi_n, \theta_n) \to (\psi, \phi, \theta)$ be uniform convergence in n and $\{\psi_n\}$ be sequences of continuous functions. Also, the pair $(\phi_n, \theta_n) \rightrightarrows (\phi, \theta)$ is two nondecreasing and continuous functions. Furthermore, let $\varphi : [0, \infty) \to [0, \infty)$ be a Lebesgue-integrable function, which is summable on each compact subset of $[0, \infty)$ and $\int_0^{\varepsilon} \varphi(t)dt > 0$ for each $\varepsilon > 0$. Moreover, assume that the following conditions hold:

i) $\theta_n(t) \rightrightarrows \theta(t) = 0$ if and only if $t = 0$ and $\theta_n \geqslant 0$ for all n,

ii) for any sequence in $\{a_k\}$ in $\mathbb{R}^+$ with $a_k \to t > 0$,

$$\psi_n (t) - \lim_{k\to\infty} sup\phi_n (a_k) + \lim_{k\to\infty} \inf \theta_n (a_k) > 0.$$

Proof. By the similar idea used in Theorem 3.1, we assume $\mu(E_k) > 0$ for all $k \geqslant 0$. By using (9), we have

$$\begin{aligned} \psi_n \left(\int_0^{\mu(E_{k+1})} \varphi(t)dt \right) &= \psi_n \int_0^{\mu(Conv(TE_k))} \varphi(t)dt \\ &= \psi_n \left(\int_0^{\mu(TE_k)} \varphi(t)dt \right) \qquad (10) \\ &\leqslant \phi_n \left(\int_0^{\mu(E_k)} \varphi(t)dt \right) - \theta_n \left(\int_0^{\mu(E_k)} \varphi(t)dt \right). \end{aligned}$$

Since $\theta_n > 0$ for all n, we have

$$\psi_n \left(\int_0^{\mu(E_{k+1})} \varphi(t)dt \right) \leqslant \phi_n \left(\int_0^{\mu(E_k)} \varphi(t)dt \right).$$

Also, from Definition 2.7, we get the following inequality

$$\int_0^{\mu(E_{k+1})} \varphi(t)dt \leqslant \int_0^{\mu(E_k)} \varphi(t)dt.$$

Thus $\left\{\int_0^{\mu(E_k)} \varphi(t)dt\right\}$ is positive but decreasing sequence. Therefore, there exists $s \geqslant 0$ such that $\lim_{k\to\infty} \int_0^{\mu(E_k)} \varphi(t)dt = s$. Since $\{\psi_n\}$ is sequence of continuous functions and shifting distance function property, also let $(\psi_n, \phi_n, \theta_n) \rightrightarrows (\psi, \phi, \theta)$, we have

$$\begin{aligned}
\lim sup_{k\to\infty}\psi\left(\int_0^{\mu(E_{k+1})} \varphi(t)dt\right) &\leqslant \lim sup_{k\to\infty}\phi\left(\int_0^{\mu(E_k)} \varphi(t)dt\right) + \lim sup_{k\to\infty} - \theta\left(\int_0^{\mu(E_k)} \varphi(t)dt\right) \\
\psi\left(\lim sup_{k\to\infty}\int_0^{\mu(E_{k+1})} \varphi(t)dt\right) &\leqslant \lim sup_{k\to\infty}\phi\left(\int_0^{\mu(E_k)} \varphi(t)dt\right) + \lim sup_{k\to\infty} - \theta\left(\int_0^{\mu(E_k)} \varphi(t)dt\right) \\
\psi\left(s\right) &\leqslant \lim sup_{k\to\infty}\phi\left(\int_0^{\mu(E_k)} \varphi(t)dt\right) + \lim \inf_{k\to\infty} \theta\left(\int_0^{\mu(E_k)} \varphi(t)dt\right).
\end{aligned}$$

Equivalently, we have

$$\psi\left(s\right) - \lim sup_{k\to\infty}\phi\left(\int_0^{\mu(E_k)} \varphi(t)dt\right) + \lim \inf_{k\to\infty} \theta\left(\int_0^{\mu(E_k)} \varphi(t)dt\right) \leqslant 0.$$

Hence $\lim_{k\to\infty} \int_0^{\mu(E_k)} \varphi(t)dt = s = 0$ and from the definition of $\varphi(t)$, we get $\lim_{k\to\infty} \mu(E_k) = 0$. As a result, we can say that T has a fixed point in C. Hence this completes the proof.

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Vatan Karakaya
Professor of Mathematics
Department of Mathematical Engineering
Yildiz Technical University Davutpasa Campus
Faculty of Chemistry Metallurgical
34210 Istanbul, Turkey
E-mail: vkkaya@yahoo.com

Derya Sekman
Ph.D Student of Mathematics
Department of Mathematics
Faculty of Science and Art
Ahi Evran University Bagbaşi Campus
40100 Kırşehir,Turkey
E-mail: deryasekman@gmail.com