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On A Generalization of Weak M-Armendariz Rings

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Abstract. For a ring R and a monoid M, we introduce J-M-Armendariz rings which are a generalization of weak M-Armendariz and J-Armendariz rings, and we investigate their properties. We prove that if $\frac{R}{J(R)}$ is a reduced ring and R is a J-M-Armendariz ring, then R is J-$M \times N$-Armendariz, where N is a unique product monoid. It is also shown that a finitely generated Abelian group G is torsion free if and only if there exists a ring R such that R is J-G-Armendariz.

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1. Introduction

Throughout this paper R denotes an associative ring with identity. For a ring R, use the symbol $T_n(R)$, $J(R)$, $Nil(R)$ to denote upper triangular matrix $n \times n$ over R, the Jacobson radical of R, and the set of all nilpotent elements of R, respectively. For a ring R, R is said to be Armendariz

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ring if for nonzero polynomials $f(x) = \sum_{i=1}^{n} a_i x^i$ and $g(x) = \sum_{j=1}^{m} b_j x^j$, $f(x)g(x) = 0$, implies that $a_i b_j = o$ for each i, j. In [1] E.Armendariz had noted that a reduced ring satisfies this condition so this name is chosen. A ring R is called weak Armendariz if nonezero polynomials $f(x) = \sum_{i=1}^{n} a_i x^i$ and $g(x) = \sum_{j=1}^{m} b_j x^j \in R[x]$ with $f(x)g(x) = 0$, implies that $a_i b_j \in Nil(R)$ for each i, j,[6]. A ring R is called J-Armendariz [10], if for any polynomials $f(x) = \sum_{i=1}^{n} a_i x^i$, $g(x) = \sum_{j=1}^{m} b_j x^j \in R[x] - \{0\}$ satisfy $f(x)g(x) = 0$, implies that $a_i b_j \in J(R)$ for each i, j. It has shown that weak Armendariz rings are J-Armendariz [10], but the converse is not true. Let M be a monoid. Liu [5] introduced another generalization of Armendariz ring which is called M-Armendariz ring, if for elements $\alpha = \Sigma_{i=1}^{n} a_i g_i$, $\beta = \Sigma_{j=1}^{m} b_j h_j \in R[M] - \{0\}$ with $\alpha\beta = 0$, implies that $a_i b_j = 0$ for each i, j. If $M = \{e\}$, then every ring is Armendariz. Zhang and Chen [11] defined a ring R, weak M-Armendariz if for two nonzero elements $\alpha = \Sigma_{i=1}^{n} a_i g_i$, $\beta = \Sigma_{j=1}^{m} b_j h_j \in R[M]$ with $\alpha\beta = 0$, implies that $a_i b_j \in Nil(R)$ for each i, j.

Motivated by the above results we introduce J-M-Armendariz rings. We define a ring R, J-M-Armendariz if whenever $\alpha = a_1 g_1 + \cdots + a_n g_n$, $\beta = b_1 h_1 + \cdots + b_m h_m \in R[M] - \{0\}$ satisfy $\alpha\beta = 0$, then $a_i b_j \in J(R)$ for each i, j.

2. Different Conditions on Monoids

We start this section by the definition of J-M-Armendariz rings and then we investigate the properties of them by different conditions on monoid.

Definition 2.1. *For a monoid M, a ring R is said to be J-M-Armendariz if whenever elements $\alpha = a_1 g_1 + \cdots + a_n g_n$, $\beta = b_1 h_1 + \cdots + b_m h_m \in R[M] - \{0\}$ satisfy $\alpha\beta = 0$, then $a_i b_j \in J(R)$ for each i, j.*

Weak M-Armendariz rings are J-M-Armendariz, because for $\alpha = a_1 g_1 + \ldots + a_n g_n, \beta = b_1 h_1 + \ldots + b_m h_m \in R[M] - \{0\}$ with $\alpha\beta = 0$, we have for each $x \in R$, $x a_i b_j \in Nil(R)$ since R is a Weak M-Armendariz ring, then $1 - x a_i b_j \in U(R)$, therefore $a_i b_j \in J(R)$. Hence R is J-M-Armendariz. Now we consider $M = (\mathbb{N} \cup \{0\}, +)$ and $D = M_3(\mathbb{Z}_2\,[[t]])$. Assume $B = \{(m_{ij}) \in D | m_{ij} \in t\mathbb{Z}_2\,[[t]]\ for\, 1 \leqslant i, j \leqslant 2\, and\, m_{ij} = 0\, for\, i = 3\ or\ j = 3\}$ and $C = \{(m_{ij}) \in D | m_{ij} = 0\ for\ i \neq j\}$. Let $R = \langle B, C \rangle$.

Therefore, R is J-M-Armendariz, but it is not weak M-Armendariz by ([10], Example 2.2) and this shows that J-M-Armendariz rings are not necessary weak M-Armendariz.

Clearly, if $\bar{R} = \frac{R}{J(R)}$ is M-Armendariz then R is J-M-Armendariz, but the following example shows that the converse is not true.

Example 2.2. Let R denote the localization of the ring $\mathbb{Z}$ of integers at the prime ideal $\langle 3 \rangle$. Consider the quaternions Q over R, that is a free R-module with basis $1, i, j, k$ and multiplication satisfying $i^2 = j^2 = k^2 = -1$, $ij = k = -ji$. Also, set $M = \mathbb{N} \cup \{0\}$. Then Q is an non-commutative domain with $J(Q) = 3Q$, and so is J-M-Armendariz. But $\dfrac{Q}{J(Q)}$ is isomorphic to the $2 - by - 2$ full matrix ring over $\mathbb{Z}_3$ and is not M-Armendariz by ([9], Remark 3.1).

Lemma 2.3. *For a ring R and a cyclic group M of order $n \geqslant 2$, R is not a J-M-Armendariz ring.*

Proof. Let $M = \{e, g, g^2, \cdots, g^{n-1}\}$. If $a = \Sigma_{k=0}^{n-1} 1.g^k$ and $b = 1e + (-1)g$. Then $ab = 0$, but 1.1 is not in $J(R)$. $\square$

Lemma 2.4. *For a monoid M and submonoid N of M, we have R is J-N-Armendariz if R is J-M-Armendariz.*

Proof. Let $\alpha = \sum_{i=1}^{n} a_i g_i$, $\beta = \sum_{j=1}^{m} b_j h_j \in R[N] - \{0\}$ such that $\alpha\beta = 0$. Since $g_i, h_j \in N \subseteq M$ for each $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant m$, then $\alpha, \beta \in R[M]$, so $a_i b_j \in J(R)$ since R is J-M-Armendariz. Therefore, R is J-N-Armendariz ring. $\square$

Theorem 2.5. *For a finitely generated Abelian group G, G is torsion-free(i.e. The set of elements of finite order in G is $\{e\}$) iff there exists J-G-Armendariz ring R with $|R| \geqslant 2$.*

Proof. If G is not torsion-free, then there exists $e \neq g \in G$, such that g has finite order. Set $N =< g >$. If $R(|R| \geqslant 2)$ is J-G-Armendariz, then R is J-N-Armendariz by Lemma 2.4. But by Lemma 2.3, R is not J-N-Armendariz, contradiction. The converse is clear by ([5],Theorem 1.14). $\square$

An element a of a monoid M is *left cancellative* if $ax = ay$ implies $x = y$ for all x, y, and is *right cancellative* if $xa = ya$ implies $x = y$

for all x, y. It is cancellative if it is both left and right cancellative. A monoid M is *cancellative* if all of its elements are.

Proposition 2.6. *For a cancellative monoid M and an ideal N of M. We have R is J-M-Armendariz if R is J-N-Armendariz.*

Proof. Let $\alpha = \Sigma_{i=1}^{n} a_i g_i$, $\beta = \Sigma_{j=1}^{m} b_j h_j$ be nonzero elements of $R[M]$ such that $\alpha\beta = 0$. Since M is cancellative, then $gg_i \neq gg_j$ and $h_i g \neq h_j g$ for $i \neq j$ and $g \in N$. Clearly, $(\sum_{i=1}^{n} a_i g g_i)\ (\sum_{j=1}^{m} b_j h_j g) = 0$ and $gg_i, h_j g \in N (1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m)$ since N is an ideal. Therefore, $a_i b_j \in J(R)$, since R is J-N-Armendariz and the proof is done. □

For a monoid M recall that a monoid M is said to be an unique product monoid (u.p.-monoid) if for any two nonempty finite subset $A, B \subseteq M$ there exists an element $g \in M$ uniquely in the form ab where $a \in A$ and $b \in B$ (A, B are finite).

Proposition 2.7. *For a u.p.-monoid M, R is J-M-Armendariz if $\bar{R} = \frac{R}{J(R)}$ is a reduced ring.*

Proof. Since $\bar{R}$ is reduced, then $\bar{R}$ is M-Armendariz by ([5], Proposition 1.1) and so R is J-M-Armendariz. □

Proposition 2.8. *For a monoid M and a u.p.-monoid N, $R[M]$ is J-N-Armendariz if $\bar{R} = \frac{R}{J(R)}$ is reduced and R is J-M-Armendariz.*

Proof. Clearly, $\bar{R}[M]$ is N-Armendariz by ([5], Proposition 2.1). Now since $\frac{R[M]}{J(R[M])} \cong \frac{R}{J(R)}[M]$, then $R[M]$ is J-N-Armendariz. □

Proposition 2.9. *For a monoid M and an u.p.-monoid N, we have $R[N]$ is J-M-Armendariz if $\bar{R} = \frac{R}{J(R)}$ is reduced and R is J-M-Armendariz.*

Proof. By Proposition 2.9, R is J-N-Armendariz and so the proof is done by ([5],Proposition 2.2). □

Proposition 2.10. *For a monoid M and an u.p.-monoid N , we have R is J-$M \times N$-Armendariz if $\bar{R} = \frac{R}{J(R)}$ is a reduced, $J(R[M]) \subseteq J(R)[M]$ and R is J-M-Armendariz.*

Proof. suppose $\sum_{i=1}^{s} a_i(m_i, n_i) \in R[M \times N]$. Without loss of generality, we assume that $\{n_1, n_2, \cdots, n_s\} = \{n_1, n_2, \cdots, n_t\}$ with $n_i \neq n_j$ when $1 \leqslant i \neq j \leqslant t$, For any $1 \leqslant p \leqslant t$, denote $A_p = \{i \mid 1 \leqslant i \leqslant s, n_i =$

$n_p\}$. Then $\sum_{p=1}^{t}(\sum_{i\in A_p} a_im_i)n_p \in R[M][N]$. Note that $m_i \neq m_p$ for any $i, i' \in A_p$ with $i \neq i'$. Now it is easy to see that exists an isomorphism of rings $R[M \times N] \longrightarrow R[M][N]$ defined by

$$\sum_{i=1}^{s} a_i(m_i, n_i) \mapsto \sum_{p=1}^{t}(\sum_{i\in A_p} a_im_i)n_p.$$

Let $(\sum_{i=1}^{s} a_i(m_i, n_i)(\sum_{j=1}^{s'} b_j(m'_j, n'_j)) = 0$ in $R[M \times N]$. So

$$(\sum_{p=1}^{t}(\sum_{i\in A_p} a_im_i)n_p)(\sum_{q=1}^{t'}(\sum_{j\in B_q} b_jm'_j)n'_j) = 0,$$

because of the above isomorphism. Hence $R[M]$ is J-M-Armendariz, by Proposition 2.9. Therefore we have $(\sum_{i\in A\in_p} a_im_i)(\sum_{j\in B_q} b_jm'_j) \in J(R[M]) \subseteq J(R)[M]$ for any p and q. Then we have $a_ib_j \in J(R)$ for any $i \in A_p$ and $j \in B_q$, since R is J-M-Armendariz. So $a_ib_j \in J(R)$ for all $i, j, 1 \leqslant i \leqslant s, 1 \leqslant j \leqslant s'$. This means that R is J-$M\times N$ -Armendariz. □

Corollary 2.11. *For a u.p.-monoid M_i, $i \in I$ and a reduced ring $\bar{R} = \frac{R}{J(R)}$, we have R is J-$\coprod_{i\in I} M_i$-Armendariz if R is J-M_{i_0}-Armendariz for some $i_0 \in I$.*

Proof. Suppose that $\alpha = \sum_i a_ig_i, \beta = \sum_j b_jh_j \in R[\coprod_{i\in I} M_i]$ with $\alpha\beta = 0$. So $\alpha, \beta \in R[M_1 \times M_2 \times \cdots, \times M_n]$ for some finite subset $\{M_1, M_2, \cdots, M_n\} \subseteq \{M_i | i \in I\}$. Therefore, $\alpha, \beta \in R[M_{i_0} \times M_1 \times \cdots \times M_n]$. The ring R, by Proposition 2.10 and induction on i, is J-$M_{i_0} \times M_1 \times \cdots \times M_n$-Armendariz. Therefore, $a_ib_j \in J(R)$ for all i and j. Hence R is J-$\coprod_{i\in I} M_i$-Armendariz. □

Proposition 2.12. *For a commutative and cancellative monoid M, we have $R[M]$ is J-Armendariz if the largest subgroup of M is $\{e\}$, $J(R)[M] \subseteq J(R[M])$, R is J-Armendariz and J-M-Armendariz.*

Proof. Let $(\sum_i \alpha_ix^i)(\sum_j \beta_jx^j) = 0$ for $\alpha_i = \sum a_{ip}g_{ip}$, $\beta_j = \sum b_{jq}h_{jq} \in R[M] - \{0\}$. If $g = (\prod_i \prod_p g_{ip})(\prod_j \prod_q h_{jq})$, then $(rh)(1g^2) = (1g^2)(rh)$ for each $h \in M$ and $r \in R$. Therefore, $(\sum_i \alpha_i(1g^2)^i)(\sum_j \beta_j(1g^2)^j) = 0$. Also $(\sum_i \sum_p a_{ip}g_{ip}g^{2i})(\sum_j \sum_q b_{jq}h_{jq}g^{2j}) = 0$. Since M is cancellative, then g_{ip} and h_{jq} are in the largest subgroup of M for each i, j, p, q. Therefore, $g_{ip} = h_{jq} = e$ by the hypothesis and then we may assume that $\alpha_i = a_i e$ and $\beta_j = b_j = e$ for all i, j. So we have $(\sum(a_i e)x^i)(\sum(b_j e)x^j) = 0$ from which it follows that $(\sum a_i x^i)(\sum b_j x^j) = 0$. Thus $a_i b_j \in J(R)$ for all i, j, since R is J-Armendariz. Hence $(a_i e)(b_j e) \in J(R)[M] \subseteq J(R[M])$. By the same discussion in the above, it follows that $(a_i e)(b_j e) \in J(R[M])$ for all i, j. Now suppose that each pair of $h_{jq}g^{2j}$'s is distinct, $a_{ip}b_{jq} \in J(R)$ for all i, p, j, q since R is J-M-Armendariz. Therefore $\alpha_i \beta_j = \sum_i \sum_j (a_{ip}b_{jq})(g_{ip}h_{jq}) = 0$. Hence $R[M]$ is J-Armendariz. □

Corollary 2.13. *For a monoid M and a reduced ring $\bar{R} = \frac{R}{J(R)}$, we have $R[x, x^{-1}]$ is J-M-Armendariz if R is J-M-Armendariz.*

Proof. Since $R[x, x^{-1}] \cong R[\mathbb{Z}]$ the proof is done. □

Proposition 2.14. *For a commutative and cancellative monoid M, we have $R[x]$ is J-Armendariz if the largest subgroup of M is $\{e\}$, $J(R)[M] \subseteq J(R[M])$, R is J-Armendariz and J-M-Armendariz.*

Proof. It is easy to see that there exists an isomorphism $R[x][M] \longrightarrow R[M][x]$ via $\sum_i(\sum_p a_{ip}x^p)g_i \longmapsto \sum_p(\sum_i a_{ip}g_i)x^p$. Hence the result follows from Proposition 2.12. □

3. Different Conditions on Rings

In this section we consider different conditions on rings and we investigate the properties of J-M-Armendariz rings.

Theorem 3.1. *For a ring R and a monoid M, let I be an ideal of R. If $\frac{R}{I}$ is J-M-Armendariz and $I \subseteq J(R)$ then R is J-M-Armendariz.*

Proof. Let $\alpha = \Sigma_{i=1}^n a_i g_i$, $\beta = \Sigma_{j=1}^m b_j h_j$ be two nonzero elements in $R[M]$ with $\alpha\beta = 0$. Therefore, $(\sum_{i=1}^n((a_i + I)g_i))(\sum_{j=1}^m((b_j + I)g_j)) = 0 \in \frac{R}{I}[M]$. Thus $(a_i+I)(b_j+I) \in J(\frac{R}{I})$ since $\frac{R}{I}$ is J-M-Armendariz. Therefore, $a_i b_j \in J(R)$, this implies that R is J-M-Armendariz. □

Proposition 3.2. *For a ring R, a monoid M and an idempotent element e of R.*

1. *If R is J-M-Armendariz, then eRe is J-M-Armendariz.*

2. *If R is an abelain ring (i.e. every idempotent element of R is central), then R is a J-M-Armendariz ring if and only if eRe is a J-M-Armendariz ring.*

Proof.

1. let $\alpha = \sum_{i=1}^{n} ea_i eg_i$, $\beta = \sum_{j=1}^{m} eb_j eh_j$ be nonzero elements of $(eRe)[M]$ such that $\alpha\beta = 0$. Since R is J-M-Armendariz and $a_i, b_j \in eRe \subseteq R$, so $a_i b_j \in J(R) \cap eRe = J(eRe)$. Therefore eRe is J-M-Armendariz.

2. One direction is clear by part 1. For converse assume that eRe is a $J-M-Armendariz$ ring and $\alpha = \sum_{i=1}^{n} a_i g_i$, $\beta = \sum_{j=1}^{m} b_j h_j$ be two nonzero elements of $R[M]$ with $\alpha\beta = 0$. Since e is a central idempotent element of R, then $0 = (e\alpha e)(e\beta e)$, where $e\alpha e, e\beta e$ are nonzero elements of $(eRe)[M]$. Therefore, $ea_i b_j e \in J(eRe) = J(R) \cap eRe$, since eRe is J-M-Armendariz ring and so R is J-M-Armendariz ring, as desired. □

Theorem 3.3. *For a ring R and a monoid M, $R[[x]]$ is J-M-Armendariz ring if and only if R is J-M-Armendariz ring.*

Proof. Suppose that R be a J-M-Armendariz ring, since $R = \frac{R[[x]]}{<x>}$ and $< x > \subseteq J(R[[x]])$, then by Theorem 3.1, $R[[x]]$ is J-M-Armendariz. For converse, let $R[[x]]$ be a J-M-Armendariz ring. Let $\alpha = \sum_{i=1}^{n} a_i g_i$, $\beta = \sum_{j=1}^{m} b_j h_j \in R[M] - \{0\}$ such that $\alpha\beta = 0$ in R. Since $R \subseteq R[[x]]$, then $\alpha\beta = 0$ in $R[[x]][M]$. By the hypothesis, we have $a_i b_j \in J(R[[x]]) = J(R) \cap R[[x]]$, so $a_i b_j \in J(R)$ for each i, j, and the proof is done. □

Theorem 3.4. *For a ring R and a monoid M, if $R[x]$ is a J-M-Armendariz ring, then R is weak M-Armendariz.*

Proof. Let $R[x]$ be a J-M-Armendariz ring, and $\alpha = \sum_{i=1}^{n} a_i g_i$, $\beta = \sum_{j=1}^{m} b_j h_j \in R[M] - \{0\}$ such that $\alpha\beta = 0$. Since $R[M] \subseteq R[x][M]$, then $\alpha\beta = 0$ in $R[x][M]$. Therefore, $a_i b_j \in J(R[x])$ for every i, j, by hypothesis. On the other hand, $J(R[x]) = I[x]$ for some nil ideal of R by ([4]). Therefore, $a_i b_j \in R \cap I[x] \subseteq Nil(R)$ and so R is weak M-Armendariz. □

Proposition 3.5. *For a ring R and a monoid M, we have the upper triangular matrix ring $T_n(R)$ is J-M-Armendariz.*

Proof. The upper triangular matrix $T_n(R)$ is weak M-Armendariz ring by ([11], Proposition 2.12). Hence it is J-M-Armendariz. $\square$

Although M-Armendariz rings for $|M| \leqslant 2$ are abelian, but Proposition 3.5 shows that J-M-Armendariz rings are not abelian in general. Now, we generalize the above Proposition to triangular ring $T = \begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$, where R and S are two rings and M is an $(R, S) - biomodule.$

Theorem 3.6. *For two rings R and S, a monoid N, an $(R, S) - biom-$ odule M and $T = \begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$. We have the rings R and S are J-N-Armendariz rings if and only if T is J-N-Armendariz ring.*

Proof. Assume that R and S are two J-N-Armendariz rings. Take $I = \begin{pmatrix} 0 & M \\ 0 & 0 \end{pmatrix}$, so $\frac{T}{I} \simeq \begin{pmatrix} R & 0 \\ 0 & S \end{pmatrix}$. Let $\alpha = \begin{pmatrix} r_1 & 0 \\ 0 & s_1 \end{pmatrix} g_1 + \cdots + \begin{pmatrix} r_n & 0 \\ 0 & s_n \end{pmatrix} g_n$ and $\beta = \begin{pmatrix} r'_1 & 0 \\ 0 & s'_1 \end{pmatrix} h_1 + \cdots + \begin{pmatrix} r'_m & 0 \\ 0 & s'_m \end{pmatrix} h_m \in \frac{T}{I}[N]$ satisfy $\alpha\beta = 0$. Define $\alpha_r = r_1 g_1 + \cdots + r_n g_n$, $\beta_r = r'_1 h_1 + \cdots + r'_m h_m \in R[N]$ and $\alpha_s = s_1 g_1 + \cdots + s_n g_n$, $\beta_s = s'_1 h_1 + \cdots + s'_m h_m \in S[N]$. From $\alpha\beta = 0$, we have $\alpha_r \beta_r = \alpha_s \beta_s = 0$. Since R and S are two $J - N - Armendariz$ rings, then we have $r_i r'_i \in J(R)$ and $s_j s'_j \in J(S)$ for any $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant m$. Since $I \subseteq J(T) = \begin{pmatrix} J(R) & M \\ 0 & J(S) \end{pmatrix} = \begin{pmatrix} 0 & M \\ 0 & 0 \end{pmatrix}$,then T is J-M-Armendariz, by Theorem 3.1. For converse, suppose that T be a J-N-Armendariz ring, $\alpha_r = r_1 g_1 + \cdots + r_n g_n$, $\beta_r = r'_1 h_1 + \cdots + r'_m h_m \in R[N]$ such that $\alpha_r \beta_r = 0$ and $\alpha_s = s_1 g_1 + \cdots + s_n g_n$, $\beta_s = s'_1 h_1 + \cdots + s'_m h_m \in S[N]$ such that $\alpha_s \beta_s = 0$. Let $\alpha = \begin{pmatrix} r_1 & 0 \\ 0 & s_1 \end{pmatrix} g_1 + \cdots + \begin{pmatrix} r_n & 0 \\ 0 & s_n \end{pmatrix} g_n$ and $\beta = \begin{pmatrix} r'_1 & 0 \\ 0 & s'_1 \end{pmatrix} h_1 + \cdots + \begin{pmatrix} r'_m & 0 \\ 0 & s'_m \end{pmatrix} h_m \in T[N]$. Whence $\alpha_r \beta_r = 0$ and $\alpha_s \beta_s = 0$ implies that $\alpha\beta = 0$. Since T is J-N-Armendariz ring then we have $\begin{pmatrix} r_i & 0 \\ 0 & s_i \end{pmatrix} \begin{pmatrix} r'_j & 0 \\ 0 & s'_j \end{pmatrix} \in J(T) = \begin{pmatrix} J(R) & M \\ 0 & J(S) \end{pmatrix}$. Then $r_i r'_j \in J(R)$ and $s_i s'_j \in J(S)$ for all i, j, as desired. $\square$

Although for a J-M-Armendariz ring R the corner ring of R is J-M-Armendariz by Proposition 3.2, but the following example shows that,

$M_n(R)$ is not J-M-Armendariz for $n \geqslant 2$, i.e. the J-M-Armendariz property is not Morita invariant.

Example 3.7. For a ring R, $S = M_2(R)$ and $M = (\mathbb{N} \cap \{0\}, +)$, if $f(x) = e_{12} - e_{11}x$ and $g(x) = e_{11} + e_{12} - (e_{21} + e_{22})x$, then $e_{11}(e_{11} + e_{12}) = e_{11} + e_{12}$ is not in $J(R)$ however, $f(x)g(x) = 0$. Thus S is not J-M-Armendariz.

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References

[1] E. P. Armrndariz, A Note on Extension of Bear and p.p.-ring, *J.Austral.Math.soc.*, 18 (1974), 470-473.

[2] G. F. Birkenmeier and J. K. Park, Triangular Matrix Representations of Ring Extensions, *Journal of Algebra*, 256 (2003), 457-477.

[3] N. K. Kim and Y. Lee, Armendariz Rings and Reduced Rings, *J. Algebra*, 2 (2000), 477-488.

[4] T. Y. Lam, *A First Course in Noncommutative Rings*, second ed., in: Graduate Texts in Mathematics, vol. 131, Springer-Verlag, New York (2001).

[5] Z. Liu, Armendariz Rings Relative to a Monoid, *Communiccations in Algebra*, 33 (3) (2006), 649-661.

[6] Z. Liu and R. Zhao, On Weak Armendariz Rings, *Comm. Algebra* , 34 (7) (2006), 2607-2616.

[7] J. Okninski, Semigroup Algebra, *New York: Marcel Dekker*, (1991).

[8] D. S. Passman, *The Algeraic Structure of Group Rings*, Wiley, New York (1977).

[9] M. B. Rege and S. Chhawchharia, Armendariz Rings, *Proc. Japan Acad. Sec. A Math. Sci.*, 73 (1) (1997), 14-17.

[10] M. Sanaee, Sh. Sahebi, and H. H. S. Javadi, J-Armendariz rings, *Journal of Mathematical Extension*, 12 (2) (2018), 65-74

[11] C. Zhang and J. Chen, Weak M-Armendariz Rings, *Southeast University*, 25 (2009), 142-146.

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