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Minimality of Pair (I, J)-Ideals in Ternary Semigroups

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Abstract. Let S be a ternary semigroup. In this article we introduce our notation and prove some elementary properties of a pair ideal (I, J) of a ternary semigroup S and give some characterizations of the minimality of pair left (right) and middle ideal in ternary semigroup.

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1. Introduction

Los [20] showed that any ternary semigroup may be embedded in an ordinary semigroup in such a way that the operation in the ternary semigroup is an (ternary) extension of the (binary) operation of the containing semigroup.

Dudek [9], Feizullaev [11], Kim and Roush [18], Lyapin [21] and Sioson [23] have also studied the properties of the ternary semigroups.

Lehmer [19] investigated certain triple systems called triplexes which turn out to be commutative ternary groups.

In 1995 and 1997, Dixit and Dewan [7, 8] introduced and studied the properties of (quasi-, bi-, left, right) lateral ideals and minimal quasi-ideals in ternary

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semigroups. In 2000, Cao and Xu [6] characterized the minimal and maximal left ideals in ordered semigroups and gave some characterizations of minimal and maximal left ideals in ordered semigroups. In 2002, Arslanov and Kehayopulu [5] characterized the minimal and maximal ideals in ordered semigroups. The concept of the minimality and maximality of (left) ideals is the really interested and important thing about (ordered) semigroups.

Also Aiyared Iampan in [16] and [17] characterize the minimality and maximality of left (right) and middle(i.e. lateral) ideals in ternary semigroups and give some characterizations of the minimality and maximality of left ideals in ternary semigroups.

In 2010, Davvaz, et al. [3, 4, 12] introduced the notion of the Γ-semihypergroup as a generalization of a semigroup, a generalization of a semihypergroup and a generalization of a Γ-semigroup.

Several characterizations and results related with generalized Γ-hyperideals, quasi-Γ-hyperideals bi-Γ-hyperideals of Γ-semihypergroups have been obtained in [1, 2, 13, 14, 15].

In this paper, we introduce the concept pair left, right and middle ideal in ternary semigroup and give some characterizations of the minimality of pair left (right) and middle ideal in ternary semigroup.

To present the main theorems we first recall the definition of a ternary semigroup which is important here.

Definition 1.1. *A nonempty set S with one ternary operation $[\,] : S\times S\times S \to S$ is called a ternary groupoid and denoted by $(S,[\,])$.*

We say that $(S,[\,])$ is a ternary semigroup if the operation $[\,]$ is associative, i.e., if

$$[[xyz]uv] = [x[yzu]v] = [xy[zuv]]$$

hold for all $x, y, z, u, v \in S$.

A non empty subset T of S is said to be a ternary subsemigroup if $T^3 = [TTT] \subset T$.

Definition 1.2. *A ternary groupoid $(S,[\,])$ is called σ-commutative,if*

$$[x_1x_2x_3] = [x_{\sigma_1}x_{\sigma_2}x_{\sigma_3}] \tag{1}$$

holds for all $x_1, x_2, x_3 \in S$ and $\sigma \in S_3$. If (1) holds for all $\sigma \in S_3$, then $(S,[\,])$ is a commutative groupoid. If (1) holds only for $\sigma = (13)$, i.e., if $[x_1x_2x_3] = [x_3x_2x_1]$, then $(S,[\,])$ is called semicommutative.

For nonempty subsets A, B and C of S, let

$$[ABC] = \{[abc] | a \in A, b \in B, c \in C\}.$$

We give the following definitions of ideals [23] as follows:

Definition 1.3. *A non empty subset I of S is said to be a*

(i) left ideal of S if $[SSI] \subseteq I$;

(ii) right ideal of S if $[ISS] \subseteq I$;

(iii) middle ideal of S if $[SIS] \subseteq I$;

(iv) two sided ideal of S if it is a left and right ideal;

(v) an ideal of S if it is a left, right and middle ideal.

An element a of a ternary semigroup S with at least two elements is called a zero element of S if $[ast] = [sat] = [sta] = a$ for all $s, t \in S$ and denote it by 0. A left ideal I of a ternary semigroup S is called a proper left ideal of S if $I \neq S$. A ternary semigroup S without zero is called left simple if it has no proper left ideals.

Definition 1.4. [7] *A ternary subsemigroup T of a ternary semigroup S is a bi-ideal of S if $[TSTST] \subseteq T$.*

Definition 1.5. [22] *An element $a \in S$ is said to be regular if there exists an element $x \in S$ such that $[axa] = a$.*

A ternary semigroup S is called regular if all of its elements are regular.

Definition 1.6. [10] *An element a in a ternary semigroup S is said to be left (resp. right) regular if there exists an element $x \in S$ such that $[xaa] = a$ (resp. $[aax] = a$).*

Definition 1.7. [10] *A subset I of a ternary semigroup S is called idempotent if $I^3 = [III] = I$.*

Lemma 1.8. [16] *For any nonempty subset A of a ternary semigroup S, $[SSASS] \cup [SAS]$ is a middle ideal of S.*

Lemma 1.9. [17] *For a nonempty subsets A of a left ideal J of a ternary semigroup S, $[JJA]$ is a left ideal of S.*

Lemma 1.10. [16] *If A is a nonempty subset of a middle ideal I of a ternary semigroup S such that $[IIAII] = [IAI]$, then $[IAI]$ is a middle ideal of S.*

2. Pair (I, J)-Ideal of Ternary Semigroups

In this section, we first introduce concept pair left, right and middle ideal of ternary semigroup and study the properties of pair left, right and middle ideals in ternary semigroups.

Definition 2.1. *Let S be a ternary semigroup and I, J two nonempty subset of S such that $I \cap J \neq \emptyset$, pair (I, J) is called to be a*

(i) pair left ideal of S if $[SIJ] \subseteq I \cap J$;

(ii) pair right ideal of S if $[IJS] \subseteq I \cap J$;

(iii) pair middle ideal of S if $[ISJ] \subseteq I \cap J$;

(iv) two side pair ideal of S if it is a pair left and pair right ideal;

(v) an pair ideal of S if it is a pair left, pair right and pair middle ideal.

A pair left (right, middle) ideal (I, J) of a ternary semigroup S is called a proper pair left (right, middle) ideal of S if $I \neq S$ or $J \neq S$.

Remark 2.2. *Let S be a ternary semigroup. Then:*

(i) If I is a middle ideal and J is a left ideal of S, then (I, J) is a pair left ideal of S.

(ii) If I is a right ideal and J is a middle ideal of S, then (I, J) is a pair right ideal of S.

(iii) If I is a right ideal and J is a left ideal of S, then (I, J) is a pair middle ideal of S.

(iv) If I is a left ideal of S, then (I, I) are pair left ideal and pair middle ideal of S.

(v) If I is a right ideal of S, then (I, I) are pair right ideal and pair middle ideal of S.

(vi) If I is a middle ideal of S, then (I, I) are pair left ideal and pair right ideal of S.

• *If S is a semicommutative and (I, J) is a pair middle ideal of S, then (J, I) is a pair middle ideal of S.*

Remark 2.3. *Let S be a ternary semigroup and T is a ternary subsemigroup of S. Then:*

(i) If (T, I) is a pair left (right, middle) ideal of S, then $T \cap I$ is a left (middle, left) ideal of T.

(ii) If (I, T) is a pair left (right, middle) ideal of S, then $T \cap I$ is a middle (right, right) ideal of T.

Proposition 2.4. *Let (I, J) be a pair left (right,middle) ideal of a ternary semigroup S and T is a ternary subsemigroup of S, then $(I \cap T, J \cap T)$ is a pair left (right,middle) ideal of T.*

Proposition 2.5. *Let (I, J) be a pair middle ideal of S. Then $I \cap J$ is a bi-ideal of S.*

Proof. Clearly that $I \cap J$ is a ternary subsemigroup of S. Hence

$$[(I \cap J)S(I \cap J)S(I \cap J)] \subseteq [[ISJ]S(I \cap J)] \subseteq [(I \cap J)S(I \cap J)] \subseteq [ISJ] \subseteq I \cap J.$$

Therefore $I \cap J$ is a bi-ideal of S. $\square$

Corollary 2.6. *Let (I, J) be a pair middle ideal of S and T is a ternary subsemigroup of S. Then $I \cap J \cap T$ is a bi-ideal of T.*

Example 2.7. Let

$$S = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}.$$

Then S is a ternary semigroup under matrix multiplication and

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

play the role of the identity in ternary semigroup S.
Take

$$I = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}$$

and

$$J = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}.$$

Then I, J not left, right and middle ideal, but (I, J) is a pair left and pair right ideal and not pair middle ideal.

Also (J, I) is not pair left, pair right and pair middle ideal.

If

$$I = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\},$$

and

$$J = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}.$$

Then I not right and middle ideal and J not left, right and middle ideal, but (I, J) is a pair left and pair right ideal and not pair middle ideal.

Also (J, I) is not pair left, pair right, but (J, I) is a pair middle ideal. Therefor

$$J \cap I = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\},$$

is a bi-idea of S.

Definition 2.8. *An element $a \in S$ is said to be quasi regular if there exists an element $x \in S$ such that $[a[axa]a] = a$.*

A ternary semigroup S is called quasi regular if all of its elements are quasi regular.

Example 2.9. Let $S = \{a, x_1, x_2\}$ such that $[xxx] = x$ for all $x \in S$ and $[xyz] = a$ if $x \neq y$ or $x \neq z$. Then S is a commutative idempotent ternary semigroup. For $I = \{a, x_1\}$ and $J = \{a, x_2\}$, (I, J) is a pair left, pair right and pair middle ideal. Also S is a regular and quasi regular.

Proposition 2.10. *Let $\{(I_\lambda, J_\lambda) | \lambda \in \Lambda\}$ be a family of pair ideals of ternary semigroup S. Then $(\cap_{\lambda\in\Lambda} I_\lambda, \cap_{\lambda\in\Lambda} J_\lambda)$ is also a pair ideals of ternary semigroup S if*

$$(\cap_{\lambda\in\Lambda} I_\lambda) \bigcap (\cap_{\lambda\in\Lambda} J_\lambda) \neq \emptyset.$$

In continue we show that if $\{(I_\lambda, J_\lambda) | \lambda \in \Lambda\}$ be a family of pair ideals of ternary semigroup S. Then $(\cup_{\lambda\in\Lambda} I_\lambda, \cup_{\lambda\in\Lambda} J_\lambda)$ is not necessary a pair ideals of ternary semigroup S.

Example 2.11. Let S be in Example 2.7. We define

$$I_1 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\},$$

and

$$J_1 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}.$$

Then (I_1, J_1) is a pair middle ideal of S. Also if

$$I_2 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\},$$

and

$$J_2 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

then (I_2, J_2) is a pair middle ideal of S, but $(I_1 \cup I_2, J_1 \cup J_2)$ is not pair middle ideal.

Lemma 2.12. *Let S be a quasi regular ternary semigroup. If (I, J) is a pair middle ideal of S, then $I \cap J$ is a regular ternary semigroup.*

Proof. For $a \in I \cap J$, there exist $x \in S$ such that $a = [a[axa]a] = [aba]$, where $b = [axa] \in I \cap J$. This implies that $I \cap J$ is a regular ternary semigroup. □

Corollary 2.13. *If (I, J) is a pair middle ideal of quasi regular ternary semigroup S, then*

$$I \cap J = [ISJ].$$

Corollary 2.14. *Let (I, J) be a pair middle ideal of quasi regular ternary semigroup S. Then $[ISJ]$ is a bi-ideal of S.*

Theorem 2.15. *Let S be a quasi regular ternary semigroup. If (I, J) is a pair right ideal and (J, K) is a pair left ideal of S, then*

$$[IJK] = I \cap J \cap K.$$

Proof. Clearly,

$$[IJK] \subseteq I \cap J \cap K.$$

Now for $a \in I \cap J \cap K$, we have $a = [a[axa]a]$ for some $x \in S$. This implies that

$$a = [a[axa]a] = [[a[axa]a][axa]a] = [[aax][aaa][xaa]] \in [IJK].$$

Thus we have $I \cap J \cap K \subseteq [IJK]$. Hence

$$[IJK] = I \cap J \cap K. \quad \square$$

Theorem 2.16. *Let S be a quasi regular ternary semigroup and (I, J) pair middle ideal. Then $I \cap J$ is idempotent.*

Proof. It is obviously,

$$(I \cap J)^3 = [(I \cap J)(I \cap J)(I \cap J)] \subset [SIJ] \subset I \cap J.$$

On the other hand for $a \in I \cap J$ there exist $x \in S$ such that $a = [a[axa]a]$. Since (I, J) pair middle ideal and $a \in I \cap J$, $[axa] \in I \cap J$. Thus $a = [a[axa]a] \in [(I \cap J)(I \cap J)(I \cap J)]$. Consequently, $I \cap J \subset [(I \cap J)(I \cap J)(I \cap J)]$ and hence $(I \cap J)^3 = [(I \cap J)(I \cap J)(I \cap J)] = I \cap J$ i.e., $I \cap J$ is idempotent. □

Corollary 2.17. *Let (I, J) be a pair middle ideal of quasi regular ternary semigroup S. Then $[ISJ]$ is idempotent.*

Definition 2.18. *A proper pair ideal (I,J) of a ternary semigroup S is called a completely semiprime of S if $x^3=[xxx]\in I\cap J$ implies that $x\in I\cap J$ for any element x of S.*

Theorem 2.19. *A ternary semigroup S is left (resp. right) regular if and only if every left (resp. right) pair ideal of S is completely semiprime.*

Proof. Let S be a left regular ternary semigroup and (I,J) be any pair left ideal of S. Suppose $a^3=[aaa]\in I\cap J$ for $a\in S$. Since S is left regular, there exists an element $x\in S$ such that $a=[xaa]=[[xx[xxx]][aaa][aaa]]\in[[SS[SSS]](I\cap J)(I\cap J)]\subseteq[SIJ]\subseteq I\cap J$. Thus (I,J) is completely semiprime.

Conversely, suppose that every pair left ideal of S is completely semiprime. Now for any $a\in S$, $(S,[Saa])$ is a pair left ideal of S. Then by hypothesis, $(S,[Saa])$ is a completely semiprime pair ideal of S. Now $a^3=[aaa]\in[Saa]$. It follows that $a\in[Saa]$. So there exists an element $x\in S$ such that $a=[xaa]$. Consequently, a is left regular. Since a is arbitrary, it follows that S is left regular.

Similarly, we can prove the theorem for right regularity. □

Lemma 2.20. *For any nonempty subset A and B of ternary semigroup S, $[SAB]$ is a left ideal of S.*

Lemma 2.21. *Let (I,J) be a pair left ideal of ternary semigroup S and A,B two subset of $I\cap J$. Then $([(I\cap J)AB],[(I\cap J)AB])$ is a pair left ideal of S.*

Proof. Since (I,J) is a pair left ideal of ternary semigroup S, then $[SIJ]\subseteq I\cap J$. Hence $[S[(I\cap J)AB][(I\cap J)AB]]\subseteq[[SSS]I[JAB]]\subseteq[[SIJ]AB]\subseteq[(I\cap J)AB]$. So $([(I\cap J)AB],[(I\cap J)AB])$ is a pair left ideal of S. □

Lemma 2.22. *Let I be a middle ideal of ternary semigroup S and J is a left ideal of S, then for any nonempty subset A of I such that $[IIAII]=[IAI]$ and for any nonempty subset B of J, $([IAI],[JJB])$ is a pair left ideal of S.*

Proof. By Lemma 1.9 and Lemma 1.10 is trivial. □

Theorem 2.23. *Let S be a ternary semigroup and (I,J) be a pair left ideal of S and let T be a left simple ternary subsemigroup of S such that $T\bigcap(I\cap J)\neq\emptyset$, then $T\subseteq(I\cap J)$.*

Proof. Since $T\bigcap(I\cap J)\neq\emptyset$, assume that $a\in T\bigcap(I\cap J)$. By Lemma 3 $[Taa]$ is a left ideal of T. Since T is a left simple, so $T=[Taa]$. Hence $T=[Taa]\subseteq[SIJ]\subseteq(I\cap J)$, so $T\subseteq(I\cap J)$. □

Theorem 2.24. *Let S be a ternary semigroup and let T be a middle simple ternary subsemigroup of S such that for a subset A of T, (A,T) is a pair left ideal of S. Then $T=A$.*

Proof. Since $A \subseteq T$, hence by Lemma 1.8 $[TTATT] \cup [TAT]$ is a middle ideal of T. Since T is a middle simple, so $T = [TTATT] \cup [TAT]$. Hence $T = [TTATT] \cup [TAT] \subseteq [S[SAT]T] \cup [SAT] \subseteq [SAT] \subseteq A$. So $T = A$. □

3. Minimal Pair Ideal

In this section, we characterize the relationship between minimality of pair left and middle ideals and left, right and middle simple ternary semigroups.

Definition 3.1. *A pair (a, a) of a ternary semigroup S is called a pair zero of S if $[aas] = [saa] = [asa] = a$ for all $s \in S$ and denote it by $(0, 0)$. A ternary semigroup S without pair zero is called pair left simple if it has no proper pair left ideals.*

A pair left (right, middle) ideal (I, J) of a ternary semigroup S without pair zero is called a minimal pair left (right, middle) ideal of S if there is no pair left (right, middle) ideal (A, B) of S such that $A \subset I$ and $B \subset J$. Equivalently, if for any pair left (right, middle) ideal (A, B) of S such that $A \subseteq I$ and $B \subseteq J$, we have $A = I$ and $B = J$.

Theorem 3.2. *If S has no zero element and (I, J) is a pair left ideal of S. Then (I, J) is a minimal pair left ideal of S such that $I \cap J$ without pair zero if and only if $I \cap J$ is pair left simple and $I = J$.*

Proof. Assume that (I, J) is a minimal pair left ideal of S such that $I \cap J$ without pair zero. Now let (A, B) be a pair left ideal of $I \cap J$. Then $[(I \cap J)AB] \subseteq A \cap B \subseteq I \cap J$. By Lemma 2.21, we have $([(I \cap J)AB], [(I \cap J)AB])$ is a pair left ideal of S. Since (I, J) is a minimal pair left ideal of S, $[(I \cap J)AB] = I = J$. Therefore $A = B = I = J$, so we conclude that $I \cap J$ is pair left simple.

Conversely, assume that $I \cap J$ is pair left simple. Let (A, B) be a pair left ideal of S such that $A \subseteq I = J$ and $B \subseteq J = I$. Then $(I \cap J) \bigcap (A \cap B) \neq \emptyset$, it follows from Theorem 2.23 that $I \cap J \subseteq A \cap B$. Hence $A = B = I = J$, so (I, J) is a minimal pair left ideal of S. □

Theorem 3.3. *If S has no zero element and I is a middle ideal of S such that for a nonempty subset A of I, $[IIAII] = [IAI]$ and (A, I) is a pair left ideal of S, and J is a left ideal of S. Then (I, J) is a minimal pair left ideal of S such that I and J without zero element if and only if I is middle simple and J is left simple.*

Proof. Assume that (I, J) is a minimal pair left ideal of S and I, J without zero element. Now let A be a middle ideal of I and let B be a left ideal of J. Then $[IAI] \subseteq A \subseteq I$ and $[JJB] \subseteq B \subseteq J$. By Lemma 2.22, we have $([IAI], [JJB])$

is a pair left ideal of S. Since (I, J) is a minimal pair left ideal of S, $[IAI] = I$ and $[JJB] = J$. Therefore $A = I$ and $B = J$, so we conclude that I is middle simple and J is left simple ideal.

Conversely, assume that I is middle simple and J is left simple. Let (A, B) be a pair left ideal of S such that $A \subseteq I$ and $B \subseteq J$. Then $A \cap I \neq \emptyset$ and $(A \cap B) \bigcap J \neq \emptyset$ it follows from Theorem 2.24 and Theorem 2.23 that $I \subseteq A$ and $J \subseteq B$ respectively. Hence $A = I$ and $B = J$, so (I, J) is a minimal pair left ideal of S. $\square$

Theorem 3.4. *Let S has no zero element and I is a right ideal, J is a left ideal of S and (I, J) is a minimal pair middle ideal of S such that I and J without zero element. Then I is right simple and J is left simple.*

Proof. Assume that (I, J) is a minimal pair middle ideal of S and I, J without zero element. Now let A be a right ideal of I and let B be a left ideal of J. Then $[AII] \subseteq A \subseteq I$ and $[JJB] \subseteq B \subseteq J$. Hence, we have $([AII], [JJB])$ is a pair middle ideal of S. Since (I, J) is a minimal pair middle ideal of S, $[AII] = I$ and $[JJB] = J$. Therefore $A = I$ and $B = J$, so we conclude that I is right simple and J is left simple ideal. $\square$

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