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Weak Soft Open Sets in Soft Bi Topological Spaces

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Abstract. This study focuses some basic result in soft bi topological spaces with respect to soft points. the results include soft limit point, soft interior point, soft neighborhood, the relation between soft weak structures and soft weak closures. Moreover the study also addresses soft sequences uniqueness of limit in soft weak-Hausdorff spaces, the product of soft Hausdorff spaces with respect to soft points in different soft weak open set and the band between soft Hausdorff space and the diagonal.

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1. Introduction

Uncertainty is inherent characteristic of modern day databases. In order to handle such databases with uncertainty, several new models have been introduced in the literature . L. A Zadeh [34] introduced some new models like fuzzy sets and Z. pawleak [41] invented rough sets and K. T. Atanassov [42] extended intuitionistic fuzzy sets. All these models have their pros and cons. However, one of the chief problems with these models is the absence of sufficient number of parameters to deal with uncertainty. In order to add adequate number of parameters. Molodtsov [45] planted the conception of soft set theory. Since then the theoretical developments on soft set theory has attracted the attention of researchers. However, the practical applications of any theory are of enough importance to make use of it.

In real life situation the problems in economics, engineering, social sciences, medical science etc. we cannot beautifully use the traditional classical methods because of different types of uncertainties and danger presented in these problems. To wash out these difficulties, some kinds of theories were put forwarded like theory of Fuzzy set, intuitionistic fuzzy set, rough set and bi polar fuzzy sets, in which we can safely use mathematical techniques for businessing with uncertainties. But, all these theories have their inherent difficulties. To overcome these difficulties in the year 1999, Russian researches Molodtsov [45] initiated the concept of soft set as a new mathematical technique to business with uncertainties. Which is free form the above difficulties. J. C. Kelly [5] studied bi-topological spaces and discussed different results.

Recently, in 2011, M. Shabir and M. Naz [6] initiated the concept of soft topological space and discussed different results with respect to ordinary points. They beautifully defined soft topology as a collection of τ of soft sets over X. They also defined the basic concept of soft topological spaces such as open set and close soft sets, soft nbhd of a point, soft separation axiom, soft regular and soft normal spaces and published their several behaviors. P. K Maji et al., [7] also discussed some fundamental results in soft topology. M. Ali et al., [8] made discussion on some new operation in soft set theory Aktas and N. Cagman [9] also studied Soft set theory Soft separation axioms are also discussed at detail. B. Chen [10] explored the parameterization reduction of soft sets and its applications. F. Feng et al., [11] studied Soft semi rings and its applications. In the recent years, many interesting applications of soft sets theory and soft topology have been discussed at great depth [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. A. M. Khattak et al., [23] planted idea over some contribution of soft regular open sets to soft separation axioms in quad soft topological spaces safely and studied some separation axioms and related results with respect to ordinary points and soft points. G. Senel [24] threw light deeply on Hausdorff space and introduced the notation of SBT points, SBT continuous functions and SBT homeomorphism and much more in soft bi topological structures.

G. Senel, N. Cagman [25] explained Soft closed sets in soft bi-topological space with great detail. G. Senel [26] studied relation between soft topological space and soft di topological space. G. Senel, N. Cagman [27] launched discussion on soft topological spaces. A. Kandil et al., [28] Soft Weak compactness that is soft semi compactness, via soft ideals are discussed with some vital results. A. Kandil et al., [29] planted the idea of soft semi (quasi) Hausdorff spaces via soft ideals. I. Zorlutana et al., [30] put remarks on soft Topological spaces.

In [31, 32, 33, 34, 35, 36] discussion is launched on soft semi open and semi closed sets, separation axioms, decomposition of some type supra soft sets and soft continuity are discussed. I. Arockiarani and A. A. Lancy [37] generalized soft g$\beta-$closed sets and soft gs$\beta-$closed sets in soft topological spaces. A. S. Mashhour et al., [38] on pre continuous and weak Precontinuous mappings. A. S. Mashhour et al [39] wrote a note on semi-continuity and pre-continuity with respect to crisp points. T. Noiri et al., [40] focused their attention on the characters of soft p-regular spaces. X. M. Naz et al., [43] beautifully produced a foot path towards Separation axiom in bi soft topological spaces. Basavaraj, M. Ittanagi [44] discussed separation axioms in more attractive way in soft bi-topological spaces. A. M. Khattak et al., [2] generalized the above idea and discussed weak separation axioms namely soft $\alpha-$separation axioms in soft bi-topological spaces with respect to ordinary points and soft points in a safe way and more over discussed hereditary properties in the same space.

A. M. Khattak et al., [47] studied some fundamental characteristics of sequences of soft real numbers. Moreover convergency and basic theorem related to soft sequences are beautifully discussed.

In this present article, the originality of the paper begin from section number 4. There in the concept of soft limit point in soft bi topological space is introduced and related results are also discussed with respect to soft points. Soft interior point in soft bi topological space and related results with respect to soft points are also studied. The direct bridge between soft weak spaces and soft weak closures is touched in soft topological space with respect to soft weak open sets. Soft neighborhood in soft bi topological space is defined and related results are studied. Soft sequences uniqueness of limit in soft weak-Hausdorff space is studied. The product of soft Hausdorff spaces with respect to soft points in different soft weak open sets are also discussed. The marriage between soft Hausdorff space and the diagonal is also been done here. We hope that these results will be valuable for the forthcoming study on soft bi topological spaces to accomplish general framework for the practical applications and to solve the most complicated problems containing doubt in economics, engineering, medical, environment and in general mechanic systems of various kinds.

2. Preliminary

The following definitions which are pre-requisites for present study

Definition 2.1. *[1] let $\mathcal{X}$ be an initial universe of discourse and $\mathcal{E}$ be the set of parameters. Let $\mathcal{P}(\mathcal{X})$ denotes the power set of $\mathcal{X}$ and $\mathcal{A}$ be a non empty subset of $\mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$is said to be a soft set over $\mathcal{X}$, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : \mathcal{A} \to \mathcal{P}(\mathcal{X})$.*

In other words, a set $\mathcal{X}$ is parameterized family of subset of universe of discourse $\mathcal{X}$. For $\xi \in \mathcal{A}$, $\mathcal{F}(\zeta)$ may be considered as the set of ξ-approximate elements of the soft set $(\mathcal{F}, \mathcal{A})$ and if $\xi \notin \mathcal{A}$ then $\mathcal{F}(\xi) = \phi$

that is $\mathcal{F}^{\mathcal{A}}=\{\mathcal{F}(\xi) : \xi \in \epsilon\mathcal{A} \subseteq \mathcal{E}, \mathcal{F} : \mathcal{A} \to \mathcal{P}(\mathcal{X})\}$ *the family of all these soft sets over* $\mathcal{X}$, *signified by* $SS(\mathcal{X})_{\mathcal{A}}$.

Definition 2.2. *[1] let* $\mathcal{F}^{\mathcal{A}}$ *and* $\mathcal{G}^{\mathcal{B}} \in SS(\mathcal{X})_{\mathcal{E}}$ *then* $\mathcal{F}^{\mathcal{A}}$ *is a soft subset of* $\mathcal{G}^{\mathcal{B}}$ *denoted by* $\mathcal{F}^{\mathcal{A}} \subseteq \mathcal{G}^{\mathcal{B}}$, *if*
1 $\mathcal{A} \subseteq \mathcal{B}$
2 $\mathcal{F}(e) \subseteq \mathcal{G}(e), \ \ \forall\ (e) \in \mathcal{A}$.
In this case $\mathcal{F}^{\mathcal{A}}$ *is said to be a soft subset of* $\mathcal{G}^{\mathcal{B}}$ *and* $\mathcal{G}^{\mathcal{B}}$ *is said to be a soft super set* $\mathcal{F}^{\mathcal{A}}, \mathcal{G}^{\mathcal{B}} \supseteq \mathcal{F}^{\mathcal{A}}$.

Definition 2.3. *[1] Two soft subsets* $\mathcal{F}^{\mathcal{A}}$ *and* $\mathcal{G}^{\mathcal{B}}$ *over a common universe of discourse set* $\mathcal{X}$, *are said to be equal if* $\mathcal{F}^{\mathcal{A}}$ *is a soft subset of* $\mathcal{G}^{\mathcal{B}}$ *and* $\mathcal{G}^{\mathcal{B}}$ *is a soft subset of* $\mathcal{F}^{\mathcal{A}}$.

Definition 2.4. *[1] The complement of soft subset* $(\mathcal{F}, \mathcal{A})$ *denoted by* $(\mathcal{F}, \mathcal{A})^c$ *is defined by* $(\mathcal{F}, \mathcal{A})^c=(\mathcal{F}^c, \mathcal{A})$ $\mathcal{F}^c : \mathcal{A} \to \mathcal{P}(\mathcal{X})$ *is a mapping given by* $\mathcal{F}^c(e)=\mathcal{X} - \mathcal{F}(e)\ \ \forall\ e \in \mathcal{A}$ *and* $\mathcal{F}^c$ *is called the soft complement function of* $\mathcal{F}$. *Clearly* $(\mathcal{F}^c)^c$ *is the same as* $\mathcal{F}$ *and* $((\mathcal{F}, \mathcal{A})^c)^c = (\mathcal{F}, \mathcal{A})$.

Definition 2.5. *[1] The difference between two soft subset* $(\psi_1, \mathcal{E})$ *and* $(\psi_2, \mathcal{E})$ *over common of universe discourse* $\mathcal{X}$ *denoted by* $(\mathcal{F}, \mathcal{E}) - (\psi_1, \mathcal{E})$ *is the soft set* $(\psi_2, \mathcal{E})$ *where for all* $e \in \mathcal{E}$.

Definition 2.6. *[1] Let* $(\mathcal{F}, \mathcal{E})$ *be a soft set over* $\mathcal{X}$ *and* $x \in \mathcal{X}$ *we say that* $x \in (\mathcal{F}, \mathcal{E})$ *and read as* x *belong to the soft set* $(\mathcal{F}, \mathcal{E})$ *whenever* $\mathcal{F}(e)\ \ \forall\ (e) \in \mathcal{E}$. *The soft set* $(\mathcal{F}, \mathcal{E})$ *over* $\mathcal{X}$ *such that* $\mathcal{F}(e) = \{x\}\ \ \forall\ (e) \in \mathcal{E}$ *is called singleton soft point and denoted by* x, *or* $(x, \mathcal{E})$.

Definition 2.7. *[1] A soft set* $(\mathcal{F}, \mathcal{A})$ *over* $\mathcal{X}$ *is said to be Null soft set denoted by* ϕ *or* ϕ_A *if* $\forall\ \ e \in \mathcal{A}, \mathcal{F}(e) = \phi$.

Definition 2.8. *[1] A soft set* $(\mathcal{F}, \mathcal{A})$ *over* $\mathcal{X}$ *is said to be an absolute soft denoted by* $\overline{\mathcal{A}}$ *or* $\mathcal{X}_A$ *if* $\forall\ \ e \in \mathcal{A}, \mathcal{F}(e) = \mathcal{X}$
Clearly, we have, ${\mathcal{X}_{\mathcal{A}}}^c = \phi_A$ *and* ${\phi_A}^c = \mathcal{X}_A$

Definition 2.9. *[1] The soft set* $(\mathcal{F}, \mathcal{A}) \in SS(\mathcal{X})_A$ *is called a soft point in* $(\mathcal{X})_A$, *denoted by* $e_{\mathcal{F}}$, *if for the element* $e \in \mathcal{A}, \mathcal{F}(e) \notin \phi_A$ *and* $\mathcal{F}(e^{'}) = \phi_A$ *if for all* $e^{'} \in A - \{e\}$.

Definition 2.10. *[1] The soft point* $e_{\mathcal{F}}$ *is said to be in the soft set* $(\mathcal{G}, \mathcal{A})$, *denoted by* $e_{\mathcal{F}} \in (\mathcal{G}, \mathcal{A})$ *if for the element* $e \in \mathcal{A}, \mathcal{F}(e) \subseteq \mathcal{G}(e)$.

Definition 2.11. *[1] Two soft sets* $(\mathcal{G}, \mathcal{A}), (\mathcal{H}, \mathcal{A})$ *in* $SS(\mathcal{X})_A$ *are said to be soft disconnected, written* $(\mathcal{F}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi_A$ *If* $\mathcal{G}(e) \cap \mathcal{H}(e) = {\phi_A}^c$ *for all* $e \in A$.

Definition 2.12. *[1] The soft point* $e_{\mathcal{G}}, e_{\mathcal{H}}$ *in* $\mathcal{X}_A$ *are disconnected, written* $e_{\mathcal{G}} \neq e_{\mathcal{H}}$ *if their corresponding soft sets* $(\mathcal{G}, \mathcal{A})$ *and* $(\mathcal{H}, \mathcal{A})$ *are disconnected.*

Definition 2.13. *[1] The union of two soft sets* $(\mathcal{F}, \mathcal{A})$ *and* $(\mathcal{G}, \mathcal{B})$ *over the common universe of discourse* $\mathcal{X}$ *is the soft set* $(\mathcal{H}, \mathcal{C})$, *where,* $\mathcal{C} = \mathcal{A} - \mathcal{B} \;\; \forall e \in \mathcal{C}$

$$\mathcal{H}(e) = \begin{cases} \mathcal{F}(e), & if \;\; e \in \mathcal{A} - \mathcal{B} \\ \mathcal{G}(e), & if \;\; e \in \mathcal{B} - \mathcal{A} \\ \mathcal{F}(e), & if \;\; e \in \mathcal{A} \cap \mathcal{B}. \end{cases}$$

Written as $(\mathcal{F}, \mathcal{A}) \cup (\mathcal{G}, \mathcal{B}) = (\mathcal{H}, \mathcal{C})$.

Definition 2.14. *[1] The intersection* $(\mathcal{H}, \mathcal{C})$ *of two soft sets* $(\mathcal{F}, \mathcal{A})$ *and* $(\mathcal{G}, \mathcal{B})$ *over common universe* $\mathcal{X}$, *denoted* $(\mathcal{F}, \mathcal{A}) \overline{\cap} (\mathcal{G}, \mathcal{B})$ *is defined as* $\mathcal{F} = \mathcal{A} \cap \mathcal{B}$ *and* $\mathcal{H}(e) = \mathcal{F}(e) \cap \mathcal{G}(e), \forall \; e \in \mathcal{C}$.

Definition 2.15. *[1] Let* $(\mathcal{F}, \mathcal{E})$ *be a soft set over* $\mathcal{X}$ *and* $\mathcal{Y}$ *be a non-empty sub set of* $\mathcal{X}$. *Then the sub soft set of* $(\mathcal{F}, \mathcal{E})$ *over* $\mathcal{Y}$ *denoted by* $(\mathcal{Y}_{\mathcal{F}}, \mathcal{E})$, *is defined as follow* $\mathcal{Y}_{\mathcal{F}}(\alpha) = \mathcal{Y} \cap \mathcal{F}(\alpha) \;\; \forall \; e \in \mathcal{E}$ *in other words,* $(\mathcal{Y}_{\mathcal{F}}, \mathcal{E}) = \mathcal{Y} \cap (\mathcal{F}, \mathcal{E})$.

Definition 2.16. *[1] Let* τ *be the collection of soft sets over* $\mathcal{X}$, *then* τ *is said to be a soft topology on* $\mathcal{X}$, *if*
1. $\phi_{\mathcal{A}}, \mathcal{X} \in \tau$
2. The union of any number of soft sets in τ *belong to* τ
3. The intersection of any two soft sets in τ *belong to* τ
The triplet $(\mathcal{X}, \tau, \mathcal{E})$ *is called a soft topological space.*

Definition 2.17. *[1] Let* $(\mathcal{X}, \tau, \mathcal{E})$ *be a soft topological space over* $\mathcal{X}$, *then the member of* τ *are said to be soft open sets in* $\mathcal{X}$.

Definition 2.18. *[1] Let* $(\mathcal{X}, \tau, \mathcal{E})$ *be a soft topological space over* $\mathcal{X}$. *A soft set* $(\mathcal{F}, \mathcal{A})$ *over* $\mathcal{X}$ *is said to be a soft closed set in* $\mathcal{X}$ *if its relative complement* $(\mathcal{F}, \mathcal{E})^c \in \tau$.

Definition 2.19. *[46] Let $(\mathcal{X},\tau,E)$ be a soft topological space and $(F,E)\subseteq SS(\mathcal{X})_{\mathcal{A}}$ then (F,E) is called α-open soft set if $(F,E)\subseteq In(Cl(In(F,E)))$. The set of all α-open soft set is denoted by $S\alpha O(\mathcal{X},\tau,E)$ or $S\alpha O(\mathcal{X})$ and the set of all α-closed soft set is denoted by $S\alpha C(\mathcal{X},\tau,E)$ or $S\alpha C\mathcal{X}$.*

Definition 2.20. *[46] Let $(\mathcal{X},\tau,E)$ be a soft topological space and $(F,E)\subseteq SS(\mathcal{X})_{\mathcal{A}}$ then (F,E) is called β-open soft set if $(F,E)\subseteq Cl(Int(Cl(F,E)))$ The set of all β-open soft set is denoted by $S\beta O(\mathcal{X},\tau,E)$ or $S\beta O(\mathcal{X})$ and the set of all β-closed soft set is denoted by $S\beta C(\mathcal{X},\tau,E)$ or $S\beta C\mathcal{X}$*

Definition 2.21. *[37] Let $(\mathcal{X},\tau,E)$ be a soft topological space and $(F,E)\subseteq SS(\mathcal{X})_{\mathcal{A}}$ then (F,E) is called pre-open soft set if $(F,E)\subseteq Int(Cl((F,E)))$ and (F,E) is called pre-close soft set if $(F,E)\supseteq Cl((Int((F,E)))$.*

3. Soft Weak Separation Axioms of Soft Topological Spaces

Definition 3.1. *[2] Let $(\mathcal{X},\tau,\mathcal{A})$ be a soft topological space over $\mathcal{X}$ and $e^g=e_{\mathcal{G}}, e^h=e_{\mathcal{H}}\in\mathcal{X}_A$ such that $e_{\mathcal{G}}\neq e_{\mathcal{H}}$. if there exists at least one soft α-open set $(\psi_1,\mathcal{A}e_{\mathcal{G}})\notin(\psi_2,\mathcal{A})$ such that $e_{\mathcal{G}}\in(\psi_1,\mathcal{A}), e_{\mathcal{H}}\notin(\psi_1,\mathcal{A})$ or $e_{\mathcal{H}}\in(\psi_2,\mathcal{A})$. Then $(\mathcal{X},\tau,\mathcal{A})$ is called a soft α_0 space.*

Definition 3.2. *[2] Let $(\mathcal{X},\tau,\mathcal{A})$ be a soft Topological spaces over $\mathcal{X}$ and $e_{\mathcal{G}}, e_{\mathcal{H}}\in\mathcal{X}_{\mathcal{A}}$ such that $e_{\mathcal{G}}\neq e_{\mathcal{H}}$ if there exists soft α-open sets $(\psi_1,\mathcal{A})$ and $(\psi_2,\mathcal{A})$ such that $e_{\mathcal{G}}\in(\psi_1,\mathcal{A}), e_{\mathcal{H}}\notin(\psi_1,\mathcal{A})$ and $e_{\mathcal{H}}\in(\psi_2,\mathcal{A}), e_{\mathcal{G}}\notin(\psi_2,\mathcal{A})$. Then $(\mathcal{X},\tau,\mathcal{A})$ is called soft α_1 space.*

Definition 3.3. *[2] Let $(\mathcal{X},\tau,\mathcal{A})$ be a soft Topological spaces over $\mathcal{X}$ and $e_{\mathcal{G}}, e_{\mathcal{H}}\in\mathcal{X}_{\mathcal{A}}$ such that $e_{\mathcal{G}}\neq e_{\mathcal{H}}$ if there exists soft α-open sets $(\psi_1,\mathcal{A})$ and $(\psi_2,\mathcal{A})$ such that $e_{\mathcal{G}}\in(\psi_1,\mathcal{A})$ and $e_{\mathcal{H}}\in(\psi_2,\mathcal{A})$
$(\psi_1,\mathcal{A})\cap(\psi_2,\mathcal{A})=\phi_{\mathcal{A}}$. Then $(\mathcal{X},\tau,\mathcal{A})$ is called soft α_2 space.*

Definition 3.4. *[4] Let $(\mathcal{X},\tau,\mathcal{A})$ be a soft Topological spaces over $\mathcal{X}$ and $e_{\mathcal{G}}, e_{\mathcal{H}}\in\mathcal{X}_{\mathcal{A}}$ such that $e_{\mathcal{G}}\neq e_{\mathcal{H}}$ if there exists at least one soft β-open sets $(\psi_1,\mathcal{A})$ or $(\psi_2,\mathcal{A})$ such that $e_{\mathcal{G}}\in(\psi_1,\mathcal{A}), e_{\mathcal{H}}\notin(\psi_1,\mathcal{A})$ or $e_{\mathcal{H}}\in(\psi_2,\mathcal{A}), e_{\mathcal{G}}\notin(\psi_2,\mathcal{A})$. Then $(\mathcal{X},\tau,\mathcal{A})$ is called soft β_0 space.*

Definition 3.5. *[4] Let* $(\mathcal{X}, \tau, \mathcal{A})$ *be a soft Topological spaces over* $\mathcal{X}$ *and* $e_{\mathcal{G}}, e_{\mathcal{H}} \in \mathcal{X}_{\mathcal{A}}$ *such that* $e_{\mathcal{G}} \neq e_{\mathcal{H}}$ *if there exists soft* β*-open sets* $(\psi_1, \mathcal{A})$ *and* $(\psi_2, \mathcal{A})$ *such that* $e_{\mathcal{G}} \in (\psi_1, \mathcal{A}), e_{\mathcal{H}} \notin (\psi_1, \mathcal{A})$ *and* $e_{\mathcal{H}} \in (\psi_2, \mathcal{A}), e_{\mathcal{G}} \notin (\psi_2, \mathcal{A})$*. Then* $(\mathcal{X}, \tau, \mathcal{A})$ *is called soft* β_1 *space.*

Definition 3.6. *[4] Let* $(\mathcal{X}, \tau, \mathcal{A})$ *be a soft Topological spaces over* $\mathcal{X}$ *and* $e_{\mathcal{G}}, e_{\mathcal{H}} \in \mathcal{X}_{\mathcal{A}}$ *such that* $e_{\mathcal{G}} \neq e_{\mathcal{H}}$ *if there exists soft* β*-open sets* $(\psi_1, \mathcal{A})$ *and* $(\psi_2, \mathcal{A})$ *such that* $e_{\mathcal{G}} \in (\psi_1, \mathcal{A})$ *and* $e_{\mathcal{H}} \in (\psi_2, \mathcal{A})$
$(\psi_1, \mathcal{A}) \cap (\psi_2, \mathcal{A}) = \phi_{\mathcal{A}}$*. Then* $(\mathcal{X}, \tau, \mathcal{A})$ *is called soft* β_2 *space.*

Definition 3.7. *[4] Let* $(\mathcal{X}, \tau, \mathcal{A})$ *be a soft Topological spaces over* $\mathcal{X}$ *and* $e_{\mathcal{G}}, e_{\mathcal{H}} \in \mathcal{X}_{\mathcal{A}}$ *such that* $e_{\mathcal{G}} \neq e_{\mathcal{H}}$ *if there exists at least one soft* P*-open sets* $(\psi_1, \mathcal{A})$ *or* $(\psi_2, \mathcal{A})$ *such that* $e_{\mathcal{G}} \in (\psi_1, \mathcal{A}), e_{\mathcal{H}} \notin (\psi_1, \mathcal{A})$ *or* $e_{\mathcal{H}} \in (\psi_2, \mathcal{A}), e_{\mathcal{G}} \notin (\psi_2, \mathcal{A})$*. Then* $(\mathcal{X}, \tau, \mathcal{A})$ *is called soft* P_0 *space.*

Definition 3.8. *[4] Let* $(\mathcal{X}, \tau, \mathcal{A})$ *be a soft Topological spaces over* $\mathcal{X}$ *and* $e_{\mathcal{G}}, e_{\mathcal{H}} \in \mathcal{X}_{\mathcal{A}}$ *such that* $e_{\mathcal{G}} \neq e_{\mathcal{H}}$ *if there exists soft* P*-open sets* $(\psi_1, \mathcal{A})$ *and* $(\psi_2, \mathcal{A})$ *such that* $e_{\mathcal{G}} \in (\psi_1, \mathcal{A}), e_{\mathcal{H}} \notin (\psi_1, \mathcal{A})$ *and* $e_{\mathcal{H}} \in (\psi_2, \mathcal{A}), e_{\mathcal{G}} \notin (\psi_2, \mathcal{A})$*. Then* $(\mathcal{X}, \tau, \mathcal{A})$ *is called soft* P_1 *space.*

Definition 3.9. *[4] Let* $(\mathcal{X}, \tau, \mathcal{A})$ *be a soft Topological spaces over* $\mathcal{X}$ *and* $e_{\mathcal{G}}, e_{\mathcal{H}} \in \mathcal{X}_{\mathcal{A}}$ *such that* $e_{\mathcal{G}} \neq e_{\mathcal{H}}$ *if there exists soft* P*-open sets* $(\psi_1, \mathcal{A})$ *and* $(\psi_2, \mathcal{A})$ *such that* $e_{\mathcal{G}} \in (\psi_1, \mathcal{A})$ *and* $e_{\mathcal{H}} \in (\psi_2, \mathcal{A})$
$(\psi_1, \mathcal{A}) \cap (\psi_2, \mathcal{A}) = \phi_{\mathcal{A}}$*. Then* $(\mathcal{X}, \tau, \mathcal{A})$ *is called soft* P_2 *space.*

4. Soft Axioms of Soft bi-Topological Spaces With Respect to Soft Points

Let $\mathcal{X}$ is an initial set and E be the non-empty set of parameter. In [43, 44] soft bi topological space over the soft set $\mathcal{X}$ is introduced. M. Naz et al., [43] introduced Soft separation axioms in soft bi-topological spaces. In this section we introduced the concept of separation axioms and weak separation axioms in soft bi-topological spaces with respect to soft points and some of its basic properties are studied and applied to different results.

Definition 4.1. *[44] Let* $(\mathcal{X}, \tau_1, \mathcal{A})$ *and* $(\mathcal{X}, \tau_2, \mathcal{A})$ *be two different soft topologies on* $\mathcal{X}$*. Then* $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$ *is called a soft bi topological space. The* *two soft topologies* $(\mathcal{X}, \tau_1, \mathcal{A})$ *and* $(\mathcal{X}, \tau_2, \mathcal{A})$ *are independently satisfy the axioms of soft topology. The members of* τ_1 *are called* τ_1 *soft open set. And complement of* τ_1 *Soft open set is called* τ_1 *soft closed set. Similarly, the member of* τ_2 *are called* τ_2 *soft open sets and the complement of* τ_2 *soft open sets are called* τ_2 *soft closed set.*

Definition 4.2. *Suppose* $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$ *be a soft topological space over* $\mathcal{X}$ *and* $(\mathcal{F}, \mathcal{A}) \subseteq \mathcal{X}$*. A point* $\mathfrak{p} \subseteq \mathcal{X}$ *is called a soft limit point of* $(\mathcal{F}, \mathcal{A})$ *if every soft* α*-open set* $(\mathcal{G}, \mathcal{A})$ *where* $(\mathcal{G}, \mathcal{A}) = (\psi_1, \mathcal{A}) \cup (\psi_2, \mathcal{A})$ *such that* $(\psi_1, \mathcal{A}) \in \tau_1$ *and* $(\psi_2, \mathcal{A}) \in \tau_2$ *congaing* $\mathfrak{p}$ *contains at least one point* $(\mathcal{G}, \mathcal{A})$ *other than* $\mathfrak{p}$ *that is* $[(\mathcal{G}, \mathcal{A}) - \mathfrak{p}] \cap (\mathcal{F}, \mathcal{A}) \neq \phi$*. The set of all soft limit point of* $(\mathcal{F}, \mathcal{A})$ *is called the derived soft set and is in short hand denoted by* $(\mathcal{F}, \mathcal{A})^d$*. Unfortunately if there exists soft open set* $(\mathcal{G}, \mathcal{A})$ *such that* $[(\mathcal{G}, \mathcal{A}) - \mathfrak{p}] \cap (\mathcal{F}, \mathcal{A}) = \phi$*. Then* $\mathfrak{p}$ *is unable to be the soft limit point of* $(\mathcal{F}, \mathcal{A})$*.*

Definition 4.3. *Let* $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$ *be a soft bi topological space* $(\mathcal{F}, \mathcal{A}) \subseteq \mathcal{X}$*. A soft point* $x \in (\mathcal{F}, \mathcal{A})$ *is called soft interior point of* $(\mathcal{F}, \mathcal{A})$ *if we can search out soft open set* $(\mathcal{G}, \mathcal{A})$ *[where* $(\mathcal{G}, \mathcal{A}) = (\mathcal{F}_1, \mathcal{A}) \cup (\mathcal{F}_2, \mathcal{A})$ *such that* $(\mathcal{F}_1, \mathcal{A}) \in \tau_1$ *and* $(\mathcal{F}_2, \mathcal{A}) \in \tau_2$*] containing* x_e *such that* $x_e \in (\mathcal{G}, \mathcal{A}) \subseteq (\mathcal{F}, \mathcal{A})$*.*

Definition 4.4. *Let* $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$ *be a soft bi topological space and* $x_e \subseteq \mathcal{X}$*. A soft sub set* $(\mathcal{F}, \mathcal{A})$ *of* $\mathcal{X}$ *is called a soft neighborhood of the soft* x_e *if we can search out soft open set* $(\mathcal{G}, \mathcal{A})$ *containing* x_e *[where* $(\mathcal{G}, \mathcal{A}) = (\mathcal{F}_1, \mathcal{A}) \cup (\mathcal{F}_2, \mathcal{A})$ *such that* $(\mathcal{F}_1, \mathcal{A}) \in \tau_1$ *and* $(\mathcal{F}_2, \mathcal{A}) \in \tau_2$*] such that* $x_e \in (\mathcal{G}, \mathcal{A}) \subseteq (\mathcal{F}, \mathcal{A})$*.*

Definition 4.5. *Suppose* $(\mathcal{X}, \tau, \mathcal{A})$ *be a soft topological space. A soft sequence* $(\mathcal{X}, \tau, \mathcal{A})$ *is said to converges to a soft point* x *if every soft weak open set* $(\mathcal{G}, \mathcal{A})$ *containing* x*, contains infinitely many terms of the soft sequence* x *that is for every soft weak open set* $(\mathcal{G}, \mathcal{A})$ *containing* x*, there exists a soft positive integer* m *such that* $x_n \in (\mathcal{G}, \mathcal{A})$ *whenever* $n \geqslant m$*.*

It is very interesting to be noted that in an arbitrary soft topological space a soft sequence may converge to more than one point. For example in a soft indiscrete space a soft sequence converges to every soft point. Thus uniqueness of the soft limit is not safe in this particular *space. So the concept of limit becomes meaningless in an arbitrary soft topological space. Soft limit enjoys itself and its behavior only in a space where uniqueness is seen. In this regards uniqueness of soft limit holds beautifully in soft weak Hausdorff spaces. The upcoming result will prove this fact at end of this article.*

Theorem 4.6. *Let $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$ be a soft bi topological space and $(\mathcal{F}, \mathcal{A}) \subseteq \mathcal{X}$ then $(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d$ is soft closed.*

Proof. Eguavilantely, we will prove $[(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$ is soft open. Let $x_e[(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$ implies $x_e[(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d$ implies $x_e \notin (\mathcal{F}, \mathcal{A})$ and $x_e \notin (\mathcal{F}, \mathcal{A})^d$. Now, $x_e \notin (\mathcal{F}, \mathcal{A})$ so by definition we can find soft open set $(\mathcal{G}, \mathcal{A})$ where $(\mathcal{G}, \mathcal{A}) = (\mathcal{F}_1, \mathcal{A}) \cup (\mathcal{F}_1, \mathcal{A})$ such that $(\mathcal{F}_1, \mathcal{A}) \in \tau_1$ and $(\mathcal{F}_2, \mathcal{A}) \in \tau_2$ such that $[(\mathcal{G}, \mathcal{A}) - \{x_e\}] \cap (\mathcal{F}, \mathcal{A}) = \phi$. This implies

$$(\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A}) = \phi \tag{1}$$

we claim that $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^d = \phi$ because if $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^d \neq \phi$ then there exists $y_e \in (\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^d$ implies $y_e \in (\mathcal{G}, \mathcal{A})$ and $y_e \in (\mathcal{G}, \mathcal{A})^d$ this implies y_e is limit point of $(\mathcal{F}, \mathcal{A})$ so according to definition so every soft open set $(\mathcal{G}, \mathcal{A})$ containg y_e intersects $(\mathcal{F}, \mathcal{A})$ in some soft point other than y_e that is $[(\mathcal{G}, \mathcal{A}) - \{y_e\}] \cap (\mathcal{F}, \mathcal{A}) \neq \phi$ which implies $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A}) \neq \phi$ but this purely contradicts results. thus $((\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A}) \cup (\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^d = \phi$. This implies $(\mathcal{G}, \mathcal{A}) \subseteq [(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$ or $x_e(\mathcal{G}, \mathcal{A}) \subseteq [(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$ this implies x_e is an interior point of $[(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$. Thus $[(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$ is soft open and consequently $(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d$ is soft closed. □

Theorem 4.7. *If $(\mathcal{F}, \mathcal{A})$ is soft sub set of be a soft bi topological space $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$, then $(\mathcal{F}, \mathcal{A})$ is soft closed iff $(\mathcal{F}, \mathcal{A})^d \subseteq (\mathcal{F}, \mathcal{A})$.*

Proof. We show $(\mathcal{F}, \mathcal{A})^d \subseteq (\mathcal{F}, \mathcal{A})$. Let $x \in (\mathcal{F}, \mathcal{A})^d$ this implies x is soft limit point of $(\mathcal{F}, \mathcal{A})$. so by definition for every soft open set $(\mathcal{G}, \mathcal{A})$ where $(\mathcal{G}, \mathcal{A}) = (\mathcal{F}_1, \mathcal{A}) \cup (\mathcal{F}_2, \mathcal{A})$ such that $(\mathcal{F}_1, \mathcal{A}) \in \tau_1$ and $(\mathcal{F}_2, \mathcal{A}) \in \tau_2$ containing x, we have

$$[(\mathcal{G}, \mathcal{A}) - \{x\}] \cap (\mathcal{F}, \mathcal{A}) \neq \phi \tag{2}$$

We are to show that $x \in (\mathcal{F}, \mathcal{A})$. suppose on contrary that $x_e \notin (\mathcal{F}, \mathcal{A})$. Then definitely $x_e \in (\mathcal{F}, \mathcal{A})^c$. Then $(\mathcal{F}, \mathcal{A})^c$ is soft open because $(\mathcal{F}, \mathcal{A})$ is soft closed. Since result (2) is true for all soft open sets $(\mathcal{G}, \mathcal{A})$ containing x. So replace $(\mathcal{G}, \mathcal{A})$ by $(\mathcal{F}, \mathcal{A})^c$ in (2), we get $[(\mathcal{F}, \mathcal{A}) - \{x\}] \cap (\mathcal{F}, \mathcal{A}) \neq \phi$. Obviously $(\mathcal{F}, \mathcal{A}) - \{x\} \subseteq (\mathcal{F}, \mathcal{A})^c$. This implies $(\mathcal{F}, \mathcal{A})^c \cap (\mathcal{F}, \mathcal{A}) \neq \phi$. But this is impossible. This impossibility is taking birth due to our wrong supposition which we made at the beginning of the problem. So we are forced to accept that $x \in (\mathcal{F}, \mathcal{A})$. That is $x_e \in (\mathcal{F}, \mathcal{A})^d$ implies $x_e \in (\mathcal{F}, \mathcal{A})$. Therefore $(\mathcal{F}, \mathcal{A})^d \subseteq (\mathcal{F}, \mathcal{A})$. Conversely let $(\mathcal{F}, \mathcal{A})^d \subseteq (\mathcal{F}, \mathcal{A})$. We are to prove that $(\mathcal{F}, \mathcal{A})$ [where $(\mathcal{F}, \mathcal{A}) = (\mathcal{F}_2, \mathcal{A}) \cup (\mathcal{F}_3, \mathcal{A})$ such that $(\mathcal{F}_2, \mathcal{A}) \in \tau_1$ and $(\mathcal{F}_3, \mathcal{A}) \in \tau_2$ such that $(\mathcal{F}_2, \mathcal{A})$ and $(\mathcal{F}_3, \mathcal{A})$ are soft closed in their respective structures] soft closed or equivalently we will prove that $(\mathcal{F}, \mathcal{A})$ is soft open. Let $x_e \in (\mathcal{F}_2, \mathcal{A})^c$ this implies $x_e \notin (\mathcal{F}_2, \mathcal{A})$ but $(\mathcal{F}, \mathcal{A})^d \subseteq (\mathcal{F}, \mathcal{A})$. so $x_e \in (\mathcal{F}, \mathcal{A})^d$ this implies x is not soft limit point of $(\mathcal{F}, \mathcal{A})$ so by definition we can search out soft open set $(\mathcal{F}_4, \mathcal{A})$ where $(\mathcal{F}_4, \mathcal{A}) = (\mathcal{F}_5, \mathcal{A}) \cup (\mathcal{F}_6, \mathcal{A})$ such that $(\mathcal{F}_5, \mathcal{A}) \in \tau_1$ and $(\mathcal{F}_6, \mathcal{A}) \in \tau_2$ such that $[(\mathcal{F}_4, \mathcal{A}) - \{x_e\}] \cap (\mathcal{F}, \mathcal{A}) = \phi$. Since $x_e \notin (\mathcal{F}, \mathcal{A})$. So $[(\mathcal{F}_4, \mathcal{A})] \cap (\mathcal{F}, \mathcal{A}) = \phi$. This implies $(\mathcal{F}_4, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^c$ or $x_e \notin (\mathcal{F}_4, \mathcal{A}) \subseteq (\mathcal{F}, \mathcal{A})^c$ implies x_e is soft interior point of $(\mathcal{F}, \mathcal{A})^c$ implies $(\mathcal{F}, \mathcal{A})^c$ is open and consequently $(\mathcal{F}, \mathcal{A})$ is soft closed. □

Theorem 4.8. *Let $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$ be a soft bi topological space and $(\mathcal{F}, \mathcal{A}) \subseteq \mathcal{X}$ then $(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d$ is soft $\alpha-$closed.*

Proof. Eguavilantely, we will prove $[(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$ is soft $\alpha-$open. Let $x_e[(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$ implies $x_e[(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d$ implies $x_e \notin (\mathcal{F}, \mathcal{A})$ and $x_e \notin (\mathcal{F}, \mathcal{A})^d$. Now, $x_e \notin (\mathcal{F}, \mathcal{A})$ so by definition we can find soft $\alpha-$open set $(\mathcal{G}, \mathcal{A})$ where $(\mathcal{G}, \mathcal{A}) = (\mathcal{F}_1, \mathcal{A}) \cup (\mathcal{F}_1, \mathcal{A})$ such that $(\mathcal{F}_1, \mathcal{A}) \in \tau_1$ and $(\mathcal{F}_2, \mathcal{A}) \in \tau_2$ such that $[(\mathcal{G}, \mathcal{A}) - \{x_e\}] \cap (\mathcal{F}, \mathcal{A}) = \phi$. This implies

$$(\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A}) = \phi \tag{3}$$

we claim that $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^d = \phi$ because if $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^d \neq \phi$ then there exists $y_e \in (\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^d$ implies $y_e \in (\mathcal{G}, \mathcal{A})$ and $y_e \in (\mathcal{G}, \mathcal{A})^d$ this implies y_e is limit point of $(\mathcal{F}, \mathcal{A})$ so according to definition so every soft $\alpha-$open set $(\mathcal{G}, \mathcal{A})$ containg y_e intersects $(\mathcal{F}, \mathcal{A})$ in some soft point other than y_e that is $[(\mathcal{G}, \mathcal{A}) - \{y_e\}] \cap (\mathcal{F}, \mathcal{A}) \neq \phi$ which implies $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A}) \neq \phi$ but this purely contradicts results. thus $((\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A}) \cup (\mathcal{G}, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^d = \phi$. This implies $(\mathcal{G}, \mathcal{A}) \subseteq [(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$ or $x_e(\mathcal{G}, \mathcal{A}) \subseteq [(\mathcal{F}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A})^d]^c$ this implies x_e is an interior point of

$[(\mathcal{F},\mathcal{A})\cup(\mathcal{F},\mathcal{A})^{d}]^{c}$. Thus $[(\mathcal{F},\mathcal{A})\cup(\mathcal{F},\mathcal{A})^{d}]^{c}$ is soft $\alpha-$open and consequently $(\mathcal{F},\mathcal{A})\cup(\mathcal{F},\mathcal{A})^{d}$ is soft $\alpha-$closed. □

Theorem 4.9. *If $(\mathcal{F},\mathcal{A})$ is soft sub set of be a soft bi topological space $(\mathcal{X},\tau_1,\tau_2,\mathcal{A})$, then $(\mathcal{F},\mathcal{A})$ is soft $\alpha-$closed iff $(\mathcal{F},\mathcal{A})^{d}\subseteq(\mathcal{F},\mathcal{A})$.*

Proof. We show $(\mathcal{F},\mathcal{A})^{d}\subseteq(\mathcal{F},\mathcal{A})$. Let $x\in(\mathcal{F},\mathcal{A})^{d}$ this implies x is soft limit point of $(\mathcal{F},\mathcal{A})$. so by definition for every soft $\alpha-$open set $(\mathcal{G},\mathcal{A})$ where $(\mathcal{G},\mathcal{A})=(\mathcal{F}_1,\mathcal{A})\cup(\mathcal{F}_2,\mathcal{A})$ such that $(\mathcal{F}_1,\mathcal{A})\in\tau_1$ and $(\mathcal{F}_2,\mathcal{A})\in\tau_2$ containing x, we have

$$[(\mathcal{G},\mathcal{A})-\{x\}]\cap(\mathcal{F},\mathcal{A})\neq\phi \tag{4}$$

We are to show that $x\in(\mathcal{F},\mathcal{A})$. suppose on contrary that $x_e\notin(\mathcal{F},\mathcal{A})$. Then definitely $x_e\in(\mathcal{F},\mathcal{A})^{c}$. Then $(\mathcal{F},\mathcal{A})^{c}$ is soft $\alpha-$open because $(\mathcal{F},\mathcal{A})$ is soft $\alpha-$closed. Since result (4) is true for all soft $\alpha-$open sets $(\mathcal{G},\mathcal{A})$ containing x. So replace $(\mathcal{G},\mathcal{A})$ by $(\mathcal{F},\mathcal{A})^{c}$ in (4), we get $[(\mathcal{F},\mathcal{A})-\{x\}]\cap(\mathcal{F},\mathcal{A})\neq\phi$. Obviously $(\mathcal{F},\mathcal{A})-\{x\}\subseteq(\mathcal{F},\mathcal{A})^{c}$. This implies $(\mathcal{F},\mathcal{A})^{c}\cap(\mathcal{F},\mathcal{A})\neq\phi$. But this is impossible. This impossibility is taking birth due to our wrong supposition which we made at the beginning of the problem. So we are forced to accept that $x\in(\mathcal{F},\mathcal{A})$. That is $x_e\in(\mathcal{F},\mathcal{A})^{d}$ implies $x_e\in(\mathcal{F},\mathcal{A})$. Therefore $(\mathcal{F},\mathcal{A})^{d}\subseteq(\mathcal{F},\mathcal{A})$. Conversely let $(\mathcal{F},\mathcal{A})^{d}\subseteq(\mathcal{F},\mathcal{A})$. We are to prove that $(\mathcal{F},\mathcal{A})$ [where $(\mathcal{F},\mathcal{A})=(\mathcal{F}_2,\mathcal{A})\cup(\mathcal{F}_3,\mathcal{A})$ such that $(\mathcal{F}_2,\mathcal{A})\in\tau_1$ and $(\mathcal{F}_3,\mathcal{A})\in\tau_2$ such that $(\mathcal{F}_2,\mathcal{A})$ and $(\mathcal{F}_3,\mathcal{A})$ are soft $\alpha-$closed in their respective structures] soft $\alpha-$closed or equivalently we will prove that $(\mathcal{F},\mathcal{A})$ is soft $\alpha-$open. Let $x_e\in(\mathcal{F}_2,\mathcal{A})^{c}$ this implies $x_e\notin(\mathcal{F}_2,\mathcal{A})$ but $(\mathcal{F},\mathcal{A})^{d}\subseteq(\mathcal{F},\mathcal{A})$. so $x_e\in(\mathcal{F},\mathcal{A})^{d}$ this implies x is not soft limit point of $(\mathcal{F},\mathcal{A})$ so by definition we can search out soft $\alpha-$open set $(\mathcal{F}_4,\mathcal{A})$ where $(\mathcal{F}_4,\mathcal{A})=(\mathcal{F}_5,\mathcal{A})\cup(\mathcal{F}_6,\mathcal{A})$ such that $(\mathcal{F}_5,\mathcal{A})\in\tau_1$ and $(\mathcal{F}_6,\mathcal{A})\in\tau_2$ such that $[(\mathcal{F}_4,\mathcal{A})-\{x_e\}]\cap(\mathcal{F},\mathcal{A})=\phi$. Since $x_e\notin(\mathcal{F},\mathcal{A})$. So $[(\mathcal{F}_4,\mathcal{A})]\cap(\mathcal{F},\mathcal{A})=\phi$. This implies $(\mathcal{F}_4,\mathcal{A})\cap(\mathcal{F},\mathcal{A})^{c}$ or $x_e\notin(\mathcal{F}_4,\mathcal{A})\subseteq(\mathcal{F},\mathcal{A})^{c}$ implies x_e is soft interior point of $(\mathcal{F},\mathcal{A})^{c}$ implies $(\mathcal{F},\mathcal{A})^{c}$ is $\alpha-$open and consequently $(\mathcal{F},\mathcal{A})$ is soft $\alpha-$closed. □

Theorem 4.10. *Let $(\mathcal{X},\tau_1,\tau_2,\mathcal{A})$ be a soft bi topological space and $(\mathcal{F},\mathcal{A})\subseteq\mathcal{X}$ then $(\mathcal{F},\mathcal{A})\cup(\mathcal{F},\mathcal{A})^{d}$ is soft $\beta-$closed.*

Proof. Eguavilantely, we will prove $[(\mathcal{F},\mathcal{A})\cup(\mathcal{F},\mathcal{A})^{d}]^{c}$ is soft $\beta-$open. Let

$x_e[(\mathcal{F},\mathcal{A})\cup(\mathcal{F},\mathcal{A})^d]^c$ implies $x_e[(\mathcal{F},\mathcal{A})\cup(\mathcal{F},\mathcal{A})^d$ implies $x_e \notin (\mathcal{F},\mathcal{A})$ and $x_e \notin (\mathcal{F},\mathcal{A})^d$. Now, $x_e \notin (\mathcal{F},\mathcal{A})$ so by definition we can find soft $\beta-$open set $(\mathcal{G},\mathcal{A})$ where $(\mathcal{G},\mathcal{A}) = (\mathcal{F}_1,\mathcal{A}) \cup (\mathcal{F}_1,\mathcal{A})$ such that $(\mathcal{F}_1,\mathcal{A}) \in \tau_1$ and $(\mathcal{F}_2,\mathcal{A}) \in \tau_2$ such that $[(\mathcal{G},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A}) = \phi$. This implies

$$(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = \phi \tag{5}$$

we claim that $(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A})^d = \phi$ because if $(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A})^d \neq \phi$ then there exists $y_e \in (\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A})^d$ implies $y_e \in (\mathcal{G},\mathcal{A})$ and $y_e \in (\mathcal{G},\mathcal{A})^d$ this implies y_e is limit point of $(\mathcal{F},\mathcal{A})$ so according to definition so every soft $\beta-$open set $(\mathcal{G},\mathcal{A})$ containing y_e intersects $(\mathcal{F},\mathcal{A})$ in some soft point other than y_e that is $[(\mathcal{G},\mathcal{A}) - \{y_e\}] \cap (\mathcal{F},\mathcal{A}) \neq \phi$ which implies $(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) \neq \phi$ but this purely contradicts results. thus $((\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) \cup (\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A})^d = \phi$. This implies $(\mathcal{G},\mathcal{A}) \subseteq [(\mathcal{F},\mathcal{A}) \cup (\mathcal{F},\mathcal{A})^d]^c$ or $x_e(\mathcal{G},\mathcal{A}) \subseteq [(\mathcal{F},\mathcal{A}) \cup (\mathcal{F},\mathcal{A})^d]^c$ this implies x_e is an interior point of $[(\mathcal{F},\mathcal{A}) \cup (\mathcal{F},\mathcal{A})^d]^c$. Thus $[(\mathcal{F},\mathcal{A}) \cup (\mathcal{F},\mathcal{A})^d]^c$ is soft $\beta-$open and consequently $(\mathcal{F},\mathcal{A}) \cup (\mathcal{F},\mathcal{A})^d$ is soft $\beta-$closed. □

Theorem 4.11. *If $(\mathcal{F},\mathcal{A})$ is soft sub set of be a soft bi topological space $(\mathcal{X},\tau_1,\tau_2,\mathcal{A})$, then $(\mathcal{F},\mathcal{A})$ is soft $\beta-$closed iff $(\mathcal{F},\mathcal{A})^d \subseteq (\mathcal{F},\mathcal{A})$.*

Proof. We show $(\mathcal{F},\mathcal{A})^d \subseteq (\mathcal{F},\mathcal{A})$. Let $x \in (\mathcal{F},\mathcal{A})^d$ this implies x is soft limit point of $(\mathcal{F},\mathcal{A})$. so by definition for every soft $\beta-$open set $(\mathcal{G},\mathcal{A})$ where $(\mathcal{G},\mathcal{A}) = (\mathcal{F}_1,\mathcal{A}) \cup (\mathcal{F}_2,\mathcal{A})$ such that $(\mathcal{F}_1,\mathcal{A}) \in \tau_1$ and $(\mathcal{F}_2,\mathcal{A}) \in \tau_2$ containing x, we have

$$[(\mathcal{G},\mathcal{A}) - \{x\}] \cap (\mathcal{F},\mathcal{A}) \neq \phi \tag{6}$$

We are to show that $x \in (\mathcal{F},\mathcal{A})$. suppose on contrary that $x_e \notin (\mathcal{F},\mathcal{A})$. Then definitely $x_e \in (\mathcal{F},\mathcal{A})^c$. Then $(\mathcal{F},\mathcal{A})^c$ is soft $\beta-$open because $(\mathcal{F},\mathcal{A})$ is soft $\beta-$closed. Since result (6) is true for all soft $\beta-$open sets $(\mathcal{G},\mathcal{A})$ containing x. So replace $(\mathcal{G},\mathcal{A})$ by $(\mathcal{F},\mathcal{A})^c$ in (6), we get $[(\mathcal{F},\mathcal{A}) - \{x\}] \cap (\mathcal{F},\mathcal{A}) \neq \phi$. Obviously $(\mathcal{F},\mathcal{A}) - \{x\} \subseteq (\mathcal{F},\mathcal{A})^c$. This implies $(\mathcal{F},\mathcal{A})^c \cap (\mathcal{F},\mathcal{A}) \neq \phi$. But this is impossible. This impossibility is taking birth due to our wrong supposition which we made at the beginning of the problem. So we are forced to accept that $x \in (\mathcal{F},\mathcal{A})$. That is $x_e \in (\mathcal{F},\mathcal{A})^d$ implies $x_e \in (\mathcal{F},\mathcal{A})$. Therefore $(\mathcal{F},\mathcal{A})^d \subseteq (\mathcal{F},\mathcal{A})$. Conversely let $(\mathcal{F},\mathcal{A})^d \subseteq (\mathcal{F},\mathcal{A})$. We are to prove that $(\mathcal{F},\mathcal{A})$ [where $(\mathcal{F},\mathcal{A}) = (\mathcal{F}_2,\mathcal{A}) \cup (\mathcal{F}_3,\mathcal{A})$ such that $(\mathcal{F}_2,\mathcal{A}) \in \tau_1$ and $(\mathcal{F}_3,\mathcal{A}) \in$

τ_2 such that $(\mathcal{F}_2, \mathcal{A})$ and $(\mathcal{F}_3, \mathcal{A})$ are soft $\beta-$closed in their respective structures] soft $\beta-$closed or equivalently we will prove that $(\mathcal{F}, \mathcal{A})$ is soft $\beta-$open. Let $x_e \in (\mathcal{F}_2, \mathcal{A})^c$ this implies $x_e \notin (\mathcal{F}_2, \mathcal{A})$ but $(\mathcal{F}, \mathcal{A})^d \subseteq (\mathcal{F}, \mathcal{A})$. so $x_e \in (\mathcal{F}, \mathcal{A})^d$ this implies x is not soft limit point of $(\mathcal{F}, \mathcal{A})$ so by definition we can search out soft $\beta-$open set $(\mathcal{F}_4, \mathcal{A})$ where $(\mathcal{F}_4, \mathcal{A}) = (\mathcal{F}_5, \mathcal{A}) \cup (\mathcal{F}_6, \mathcal{A})$ such that $(\mathcal{F}_5, \mathcal{A}) \in \tau_1$ and $(\mathcal{F}_6, \mathcal{A}) \in \tau_2$ such that $[(\mathcal{F}_4, \mathcal{A}) - \{x_e\}] \cap (\mathcal{F}, \mathcal{A}) = \phi$. Since $x_e \notin (\mathcal{F}, \mathcal{A})$. So $[(\mathcal{F}_4, \mathcal{A})] \cap (\mathcal{F}, \mathcal{A}) = \phi$. This implies $(\mathcal{F}_4, \mathcal{A}) \cap (\mathcal{F}, \mathcal{A})^c$ or $x_e \notin (\mathcal{F}_4, \mathcal{A}) \subseteq (\mathcal{F}, \mathcal{A})^c$ implies x_e is soft interior point of $(\mathcal{F}, \mathcal{A})^c$ implies $(\mathcal{F}, \mathcal{A})^c$ is $\beta-$open and consequently $(\mathcal{F}, \mathcal{A})$ is soft $\beta-$closed. □

Theorem 4.12. *Soft topological space* $(\mathcal{X}, \tau_1, \mathcal{A})$, *is said to be soft* αT_0 *space iff for any two soft distinct points* $x_e, y_e \in \mathcal{X}$ *are with distinct soft* $\alpha-$*closure.*

Proof. Given Soft topological space $(\mathcal{X}, \tau_1, \mathcal{A})$, is said to be soft αT_0 space. So according to definition for every $x_e, y_e \in \mathcal{X}$ such that $x_e \neq y_e$ if there exist soft $\alpha-$open sets $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ such that $x_e \in (\mathcal{G}, \mathcal{A})$, $y_e \notin (\mathcal{G}, \mathcal{A})$ or $y_e \in (\mathcal{H}, \mathcal{A})$, $x_e \notin (\mathcal{H}, \mathcal{A})$. Now $x_e \in (\mathcal{G}, \mathcal{A})$, $y_e \notin (\mathcal{G}, \mathcal{A})$ this implies $x_e \in (\mathcal{G}, \mathcal{A})$, $y_e \notin (\mathcal{G}, \mathcal{A}) - \{x_e\}$ this implies that $(\mathcal{G}, \mathcal{A}) - \{x\} \cap \{y_e\} = \phi$ this implies x is not soft limit point of $\{y\}$ this implies $x \notin \{y\}^d$ also $x_e \notin \{y_e\}^d$ this implies $x_e \notin \{y_e\} \cup \{y_e\}^d$ this implies $x_e \notin \overline{\{y_e\}}$..(1). since $\overline{\{x_e\}} = \{x_e\} \cup \{x_e\}^d$ this implies $\overline{\{x\}} \notin \{x\}$..(2). From these two result we can at once conclude that $\overline{\{x_e\}} \neq \overline{\{y_e\}}$. Conversely let us suppose that $\overline{\{x_e\}} \neq \overline{\{y_e\}}$ for $x_e, y_e \in \mathcal{X}$ such that $x_e \neq y_e$. We have to prove that $(\mathcal{X}, \tau_1, \mathcal{A})$, is soft αT_0 space. Suppose on contrary that is $(\mathcal{X}, \tau_1, \mathcal{A})$, is not soft αT_0 space. Then every soft $\alpha-$open set $(\mathcal{G}, \mathcal{A})$ containing x also contains y that is $x_e \in (\mathcal{G}, \mathcal{A})$ and $y_e \in (\mathcal{G}, \mathcal{A})$ this implies $x_e \in (\mathcal{G}, \mathcal{A}) - \{y_e\}$ and $y_e \in (\mathcal{G}, \mathcal{A}) - \{x_e\}$ this implies $((\mathcal{G}, \mathcal{A}) - \{x_e\}) \cap \{y\} \neq \phi$ and $((\mathcal{G}, \mathcal{A}) - \{y_e\}) \cap \{x_e\} \neq \phi$. This implies x is soft limit point of $\{y_e\}$ and y is soft limit point of $\{x_e\}$ this implies $x_e \in \{y_e\}^d$ and $y_e \in \{x_e\}^d$ this implies $x_e \in \{y_e\} \cup \{y_e\}^d$ and $y_e \in \{x_e\} \cup \{x_e\}^d$ this implies $x_e \in \overline{\{y_e\}}$ and $y_e \in \overline{\{x_e\}}$ but $x_e \in \overline{\{x_e\}}$ and $y_e \in \overline{\{y_e\}}$ are obvious this implies that $\overline{\{x_e\}} = \overline{\{y_e\}}$. But this is purely contradiction to the hypothesis that $\overline{\{x_e\}} \neq \overline{\{y_e\}}$. This contradiction arises due to our wrong supposition that $(\mathcal{X}, \tau_1, \mathcal{A})$, is not soft αT_0 space. Thus $(\mathcal{X}, \tau_1, \mathcal{A})$, is soft αT_0 space. This completes the proof. □

Theorem 4.13. *Soft topological space $(\mathcal{X}, \tau_1, \mathcal{A})$, is said to be soft βT_0 space iff for any two soft distinct points $x_e, y_e \in \mathcal{X}$ are with distinct soft $\beta-$closure.*

Proof. Given Soft topological space $(\mathcal{X}, \tau_1, \mathcal{A})$, is said to be soft βT_0 space. So according to definition for every $x_e, y_e \in \mathcal{X}$ such that $x_e \neq y_e$ if there exist soft $\beta-$open sets $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ such that $x_e \in (\mathcal{G}, \mathcal{A})$, $y_e \notin (\mathcal{G}, \mathcal{A})$ or $y_e \in (\mathcal{H}, \mathcal{A})$, $x_e \notin (\mathcal{H}, \mathcal{A})$. Now $x_e \in (\mathcal{G}, \mathcal{A})$, $y_e \notin (\mathcal{G}, \mathcal{A})$ this implies $x_e \in (\mathcal{G}, \mathcal{A})$, $y_e \notin (\mathcal{G}, \mathcal{A}) - \{x_e\}$ this implies that $(\mathcal{G}, \mathcal{A}) - \{x\} \cap \{y_e\} = \phi$ this implies x is not soft limit point of $\{y\}$ this implies $x \notin \{y\}^d$ also $x_e \notin \{y_e\}^d$ this implies $x_e \notin \{y_e\} \cup \{y_e\}^d$ this implies $x_e \notin \overline{\{y_e\}}$..(1). since $\overline{\{x_e\}} = \{x_e\} \cup \{x_e\}^d$ this implies $\overline{\{x\}} \notin \overline{\{x\}}$..(2). From these two result we can at once conclude that $\overline{\{x_e\}} \neq \overline{\{y_e\}}$. Conversely let us suppose that $\overline{\{x_e\}} \neq \overline{\{y_e\}}$ for $x_e, y_e \in \mathcal{X}$ such that $x_e \neq y_e$. We have to prove that $(\mathcal{X}, \tau_1, \mathcal{A})$, is soft βT_0 space. Suppose on contrary that is $(\mathcal{X}, \tau_1, \mathcal{A})$, is not soft βT_0 space. Then every soft $\beta-$open set $(\mathcal{G}, \mathcal{A})$ containing x also contains y that is $x_e \in (\mathcal{G}, \mathcal{A})$ and $y_e \in (\mathcal{G}, \mathcal{A})$ this implies $x_e \in (\mathcal{G}, \mathcal{A}) - \{y_e\}$ and $y_e \in (\mathcal{G}, \mathcal{A}) - \{x_e\}$ this implies $((\mathcal{G}, \mathcal{A}) - \{x_e\}) \cap \{y\} \neq \phi$ and $((\mathcal{G}, \mathcal{A}) - \{y_e\}) \cap \{x_e\} \neq \phi$. This implies x is soft limit point of $\{y_e\}$ and y is soft limit point of $\{x_e\}$ this implies $x_e \in \{y_e\}^d$ and $y_e \in \{x_e\}^d$ this implies $x_e \in \{y_e\} \cup \{y_e\}^d$ and $y_e \in \{x_e\} \cup \{x_e\}^d$ this implies $x_e \in \overline{\{y_e\}}$ and $y_e \in \overline{\{x_e\}}$ but $x_e \in \overline{\{x_e\}}$ and $y_e \in \overline{\{y_e\}}$ are obvious this implies that $\overline{\{x_e\}} = \overline{\{y_e\}}$. But this is purely contradiction to the hypothesis that $\overline{\{x_e\}} \neq \overline{\{y_e\}}$. This contradiction arises due to our wrong supposition that $(\mathcal{X}, \tau_1, \mathcal{A})$, is not soft βT_0 space. Thus $(\mathcal{X}, \tau_1, \mathcal{A})$, is soft βT_0 space. This completes the proof. □

Theorem 4.14. *If $(\mathcal{F}, \mathcal{A})$ and $(\mathcal{G}, \mathcal{A})$ be soft sub sets of soft bi-topological space $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$*
(i) $((\mathcal{F}, \mathcal{A})^0)^0 = (\mathcal{F}, \mathcal{A})^0$
(ii) $(\mathcal{F}, \mathcal{A}) \subseteq (\mathcal{G}, \mathcal{A}) \Rightarrow (\mathcal{F}, \mathcal{A})^0 \subseteq (\mathcal{G}, \mathcal{A})^0$
(iii) $[(\mathcal{F}, \mathcal{A}) \cup (\mathcal{G}, \mathcal{A})]^0 \supseteq (\mathcal{F}, \mathcal{A})^0 \cup (\mathcal{G}, \mathcal{A})^0$
(iv) $[(\mathcal{F}, \mathcal{A}) \cap (\mathcal{G}, \mathcal{A})]^0 = (\mathcal{F}, \mathcal{A})^0 \cap (\mathcal{G}, \mathcal{A})^0$.

Proof. (i) Let $(\mathcal{F}, \mathcal{A})^0 = (\mathcal{G}, \mathcal{A})$ then $(\mathcal{G}, \mathcal{A})$ is soft open [where $(\mathcal{G}, \mathcal{A}) = (\mathcal{F}_2, \mathcal{A}) \cup (\mathcal{F}_3, \mathcal{A})$ such that $(\mathcal{F}_2, \mathcal{A}) \in \tau_1$ and $(\mathcal{F}_3, \mathcal{A}) \in \tau_2$ furthermore since the soft interior of a soft is always soft open. Therefore $(\mathcal{G}, \mathcal{A})]^0 = (\mathcal{G}, \mathcal{A})]$ or $((\mathcal{F}, \mathcal{A})^0)^0 = (\mathcal{F}, \mathcal{A})^0$

(ii) to prove $(\mathcal{F},\mathcal{A}) \subseteq (\mathcal{G},\mathcal{A})$ implies $(\mathcal{F},\mathcal{A})^0 \subseteq (\mathcal{G},\mathcal{A})^0$ we proceeds as fallows let $x_e \in (\mathcal{F},\mathcal{A})^0$ this means that x_e is soft interior point of $(\mathcal{F},\mathcal{A})$ so according to definition there exists soft open set $(\mathcal{H},\mathcal{A})$ where $(\mathcal{H},\mathcal{A}) = (\mathcal{F}_2,\mathcal{A}) \cup (\mathcal{F}_3,\mathcal{A})$ such that $(\mathcal{F}_2,\mathcal{A}) \in \tau_1$ and $(\mathcal{F}_3,\mathcal{A}) \in \tau_2$ containing x_e such that $x_e \in (\mathcal{H},\mathcal{A}) \subseteq (\mathcal{F},\mathcal{A})($ but $(\mathcal{F},\mathcal{A}) \subseteq (\mathcal{G},\mathcal{A})$ this implies that $x_e \in (\mathcal{H},\mathcal{A}) \subseteq (\mathcal{G},\mathcal{A})$ this implies that x_e is soft interior point of $(\mathcal{G},\mathcal{A})$ this means that $x_e \in (\mathcal{H},\mathcal{A})^0$. Hence $(\mathcal{F},\mathcal{A}) \subseteq (\mathcal{G},\mathcal{A})$ implies $(\mathcal{F},\mathcal{A})^0 \subseteq (\mathcal{G},\mathcal{A})^0$

next to prove

(iii) $(\mathcal{F},\mathcal{A})^0 \cup (\mathcal{G},\mathcal{A})^0 \supseteq (\mathcal{F},\mathcal{A})^0 \cup (\mathcal{G},\mathcal{A})^0$ for this let $x_e \in (\mathcal{F},\mathcal{A})^0 \cup (\mathcal{G},\mathcal{A})^0$ this means that $x_e \in (\mathcal{F},\mathcal{A})^0$ or $x_e \in (\mathcal{G},\mathcal{A})^0$ this means that x_e is soft interior point of $(\mathcal{F},\mathcal{A})$ so according to definition there exists soft open set $(\mathcal{H},\mathcal{A})$ where $(\mathcal{H},\mathcal{A}) = (\mathcal{F}_2,\mathcal{A}) \cup (\mathcal{F}_3,\mathcal{A})$ such that $(\mathcal{F}_2,\mathcal{A}) \in \tau_1$ and $(\mathcal{F}_3,\mathcal{A}) \in \tau_2$ containing x_e such that $x_e \in (\mathcal{H},\mathcal{A}) \subseteq (\mathcal{F},\mathcal{A})$ and this means that x_e is soft interior point of $(\mathcal{G},\mathcal{A})$ so according to definition there exists soft open set $(\mathcal{L},\mathcal{A})$ where $(\mathcal{L},\mathcal{A}) = (\mathcal{F}_4,\mathcal{A}) \cup (\mathcal{F}_5,\mathcal{A})$ such that $(\mathcal{F}_4,\mathcal{A}) \in \tau_1$ and $(\mathcal{F}_5,\mathcal{A}) \in \tau_2$ containing x_e such that $x_e \in (\mathcal{L},\mathcal{A}) \subseteq (\mathcal{G},\mathcal{A})$ mixing the above sentences we have $x_e \in (\mathcal{H},\mathcal{A}) \cup (\mathcal{L},\mathcal{A}) \subseteq (\mathcal{F},\mathcal{A}) \cup (\mathcal{G},\mathcal{A})$ this means that x_e is soft interior point of $(\mathcal{F},\mathcal{A}) \cup (\mathcal{G},\mathcal{A})$ that is $x_e \in (\mathcal{F},\mathcal{A})^0 \cup (\mathcal{G},\mathcal{A})^0$ we at once see that $[(\mathcal{F},\mathcal{A}) \cup (\mathcal{G},\mathcal{A})]^0 \supseteq (\mathcal{F},\mathcal{A})^0 \cup (\mathcal{G},\mathcal{A})^0$

next we prove

(iv) $[(\mathcal{F},\mathcal{A}) \cap (\mathcal{G},\mathcal{A})]^0 = (\mathcal{F},\mathcal{A})^0 \cap (\mathcal{G},\mathcal{A})^0$ for this we proceeds as fallows. Since $(\mathcal{F},\mathcal{A}) \cap (\mathcal{G},\mathcal{A}) \subseteq (\mathcal{F},\mathcal{A})$ and $(\mathcal{F},\mathcal{A}) \cap (\mathcal{G},\mathcal{A}) \subseteq (\mathcal{G},\mathcal{A})$ this implies that $[(\mathcal{F},\mathcal{A}) \cup (\mathcal{G},\mathcal{A})]^0 \supseteq (\mathcal{F},\mathcal{A})^0$ and $[(\mathcal{F},\mathcal{A}) \cup (\mathcal{G},\mathcal{A})]^0 \supseteq (\mathcal{G},\mathcal{A})^0$ mixing the above statements we $[(\mathcal{F},\mathcal{A}) \cap (\mathcal{G},\mathcal{A})]^0 \subseteq (\mathcal{F},\mathcal{A})^0 \cap (\mathcal{G},\mathcal{A})^0$...(1) to get the require results we have to prove that $(\mathcal{F},\mathcal{A})^0 \cap (\mathcal{G},\mathcal{A})^0 \subseteq [(\mathcal{F},\mathcal{A}) \cap (\mathcal{G},\mathcal{A})]^0$ for this we proceeds as fallows $x_e \in (\mathcal{F},\mathcal{A})^0 \cap (\mathcal{G},\mathcal{A})^0$ this means that $x_e \in (\mathcal{F},\mathcal{A})^0$ and $x_e \in (\mathcal{G},\mathcal{A})^0$ So according to definition there exists soft open set $(\mathcal{H},\mathcal{A})$ where $(\mathcal{H},\mathcal{A}) = (\mathcal{F}_2,\mathcal{A}) \cup (\mathcal{F}_3,\mathcal{A})$ such that $(\mathcal{F}_2,\mathcal{A}) \in \tau_1$ and $(\mathcal{F}_3,\mathcal{A}) \in \tau_2$ containing x_e such that $x_e \in (\mathcal{H},\mathcal{A}) \subseteq (\mathcal{F},\mathcal{A})$ and this means that x_e is soft interior point of $(\mathcal{G},\mathcal{A})$ so according to definition there exists soft open set $(\mathcal{L},\mathcal{A})$ where $(\mathcal{L},\mathcal{A}) = (\mathcal{F}_4,\mathcal{A}) \cup (\mathcal{F}_5,\mathcal{A})$ such that $(\mathcal{F}_4,\mathcal{A}) \in \tau_1$ and $(\mathcal{F}_5,\mathcal{A}) \in \tau_2$ containing x_e such that $x_e \in (\mathcal{L},\mathcal{A}) \subseteq (\mathcal{G},\mathcal{A})$ mixing the above sentences we have $x_e \in (\mathcal{H},\mathcal{A}) \cap (\mathcal{L},\mathcal{A}) \subseteq (\mathcal{F},\mathcal{A}) \cap (\mathcal{G},\mathcal{A})$ this means that x_e is soft interior point of $(\mathcal{F},\mathcal{A})\cap(\mathcal{G},\mathcal{A})$ that is $x_e\in(\mathcal{F},\mathcal{A})^0\cap(\mathcal{G},\mathcal{A})^0$ we at once see that $[(\mathcal{F},\mathcal{A})\cap(\mathcal{G},\mathcal{A})]^0=(\mathcal{F},\mathcal{A})^0\cap(\mathcal{G},\mathcal{A})^0$. □

Theorem 4.15. *Let $(\mathcal{X},\tau_1,\tau_2,\mathcal{A})$ be a soft bi topological space and $(\gamma,\mathcal{A})$ and $(\phi,\mathcal{A})$ are the soft nbhds of the soft point $x_e\in\mathcal{X}$ then so is*
(i) $(\gamma,\mathcal{A}\cap(\phi,\mathcal{A})$
(ii) $(\gamma,\mathcal{A})\cup(\phi,\mathcal{A})$.

Proof. Since $(\gamma,\mathcal{A})$ and $(\phi,\mathcal{A})$ are the soft nbhd of the soft point $x_e\in\mathcal{X}$ so according to definition we can find soft open sets $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ [where $(\mathcal{G},\mathcal{A})=(\mathcal{F}_1,\mathcal{A})\cup(\mathcal{F}_2,\mathcal{A})$ such that $(\mathcal{F}_4,\mathcal{A})\in\tau_1$ and $(\mathcal{F}_5,\mathcal{A})\in\tau_2$] and [where $(\mathcal{H},\mathcal{A})=(\mathcal{F}_3,\mathcal{A})\cup(\mathcal{F}_4,\mathcal{A})$ such that $(\mathcal{F}_3,\mathcal{A})\in\tau_1$ and $(\mathcal{F}_4,\mathcal{A})\in\tau_2$] such that $x_e\in(\mathcal{G},\mathcal{A})\subseteq(\gamma,\mathcal{A})$ and $x_e\in(\mathcal{H},\mathcal{A})\subseteq(\phi,\mathcal{A})$ maxing these results we have
(i) $x_e\in(\mathcal{G},\mathcal{A})\cap(\mathcal{H},\mathcal{A})\subseteq(\gamma,\mathcal{A})\cap(\phi,\mathcal{A})$. Also
(ii) $x_e\in(\mathcal{G},\mathcal{A})\cup(\mathcal{H},\mathcal{A})\subseteq(\gamma,\mathcal{A})\cup(\phi,\mathcal{A})$. This finishes the proof. □

Theorem 4.16. *A soft topological space $(\mathcal{X},\tau,\mathcal{A})$ is a Hausdorff space iff for every two distinct soft points x_e and y_e belong to $\mathcal{X}$, There exists soft closed sets $(\mathcal{Q},\mathcal{A})$ and $(\mathcal{F},\mathcal{A})$ such that $x_e\in(\mathcal{Q},\mathcal{A})$, $y_e\notin(\mathcal{Q},\mathcal{A})$ and $y_e\in(\mathcal{F},\mathcal{A})$, $x_e\notin(\mathcal{F},\mathcal{A})$ and $(\mathcal{Q},\mathcal{A})\cup(\mathcal{F},\mathcal{A})=\mathcal{X}$.*

Proof. Suppose $(\mathcal{X},\tau,\mathcal{A})$ be a soft Hausdorff space Then according to definition for $x_e,y_e\in\mathcal{X}$ such that $x_e\neq y_e$. If we can find two soft open sets such that $x_e\in(\mathcal{G},\mathcal{A})$ and $y_e\in(\mathcal{H},\mathcal{A})$ moreover $(\mathcal{G},\mathcal{A})\cap(\mathcal{H},\mathcal{A})=\phi$. Since $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ are mutually exclusive. So $x_e\in(\mathcal{G},\mathcal{A})$ and $y_e\in(\mathcal{H},\mathcal{A})$ implies $x_e\in(\mathcal{G},\mathcal{A}),y_e\notin(\mathcal{G},\mathcal{A})$ and $y_e\in(\mathcal{H},\mathcal{A}),x_e\notin(\mathcal{H},\mathcal{A})$ this implies $x_e\notin(\mathcal{G},\mathcal{A})^c$, $y_e\in(\mathcal{G},\mathcal{A})^c$ and $y_e\notin(\mathcal{H},\mathcal{A})^c,x_e\in(\mathcal{H},\mathcal{A})^c$ or $x_e\in(\mathcal{H},\mathcal{A})^c,y_e\notin(\mathcal{H},\mathcal{A})^c$ and $y_e\in(\mathcal{G},\mathcal{A})^c,x_e\notin(\mathcal{G},\mathcal{A})^c$.

Let $(\mathcal{Q},\mathcal{A})=(\mathcal{H},\mathcal{A})^c$ and $(\mathcal{F},\mathcal{A})=(\mathcal{G},\mathcal{A})^c$, Then $(\mathcal{Q},\mathcal{A})$ and $(\mathcal{F},\mathcal{A})$ are soft closed sets and $x_e\in(\mathcal{Q},\mathcal{A})$, $y_e\notin(\mathcal{Q},\mathcal{A})$ and $y_e\in(\mathcal{F},\mathcal{A})$, $x_e\notin(\mathcal{F},\mathcal{A})$. Also $(\mathcal{Q},\mathcal{A})\cup(\mathcal{F},\mathcal{A})=(\mathcal{G},\mathcal{A})^c\cup(\mathcal{H},\mathcal{A})^c=[(\mathcal{G},\mathcal{A})\cap(\mathcal{H},\mathcal{A})]^c=\phi^c=\mathcal{X}$. Conversely, let us suppose that for every two soft distinct points x_e and y_e of $\mathcal{X}$, there exists soft closed sets $(\mathcal{Q},\mathcal{A})$ and $(\mathcal{F},\mathcal{A})$ such that $x_e\in(\mathcal{Q},\mathcal{A})$, $y_e\notin(\mathcal{Q},\mathcal{A})$ and $y_e\in(\mathcal{F},\mathcal{A})$, $x_e\notin(\mathcal{F},\mathcal{A})$ and $(\mathcal{Q},\mathcal{A})\cup(\mathcal{F},\mathcal{A})=\mathcal{X}$. We prove that $(\mathcal{X},\tau,\mathcal{A})$ be a soft Hausdorff space. We proceeds as fallow to catch the required space. $(\mathcal{G},\mathcal{A})=(\mathcal{Q},\mathcal{A})^c$ and $(\mathcal{H},\mathcal{A})=(\mathcal{F},\mathcal{A})^c$ then $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ are soft open sets and $x_e \in (\mathcal{Q},\mathcal{A})$, $y_e \notin (\mathcal{Q},\mathcal{A})$ and $y_e \in (\mathcal{F},\mathcal{A})$, $x_e \notin (\mathcal{F},\mathcal{A})$. This implies, $y_e \notin (\mathcal{Q},\mathcal{A})$ and $x_e \notin (\mathcal{F},\mathcal{A})$ implies $y_e \in (\mathcal{Q},\mathcal{A})^c$ and $x_e \in (\mathcal{F},\mathcal{A})^c$ implies that $x_e \in (\mathcal{H},\mathcal{A})$ and $y_e \in (\mathcal{G},\mathcal{A})$. Moreover, $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = (\mathcal{F},\mathcal{A})^c \cap (\mathcal{Q},\mathcal{A})^c = [(\mathcal{F},\mathcal{A}) \cap (\mathcal{Q},\mathcal{A})]^c = \mathcal{X}^c = \phi$. This proves that for every two soft distinct points x_e and y_e of $\mathcal{X}$, there exists soft open sets $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ such hat $x_e \in (\mathcal{G},\mathcal{A})$ and $y_e \in (\mathcal{H},\mathcal{A})$ such that $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = \phi$. This completes the proof. □

Theorem 4.17. *A soft topological space $(\mathcal{X},\tau,\mathcal{A})$ is a $\alpha-$Hausdorff space iff for every two distinct soft points x_e and y_e belong to $\mathcal{X}$, There exists soft $\alpha-$closed sets $(\mathcal{Q},\mathcal{A})$ and $(\mathcal{F},\mathcal{A})$ such that $x_e \in (\mathcal{Q},\mathcal{A})$, $y_e \notin (\mathcal{Q},\mathcal{A})$ and $y_e \in (\mathcal{F},\mathcal{A})$, $x_e \notin (\mathcal{F},\mathcal{A})$ and $(\mathcal{Q},\mathcal{A}) \cup (\mathcal{F},\mathcal{A}) = \mathcal{X}$.*

Proof. Suppose $(\mathcal{X},\tau,\mathcal{A})$ be a soft $\alpha-$Hausdorff space Then according to definition for $x_e, y_e \in \mathcal{X}$ such that $x_e \neq y_e$. If we can find two soft $\alpha-$open sets such that $x_e \in (\mathcal{G},\mathcal{A})$ and $y_e \in (\mathcal{H},\mathcal{A})$ moreover $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = \phi$. Since $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ are mutually exclusive. So $x_e \in (\mathcal{G},\mathcal{A})$ and $y_e \in (\mathcal{H},\mathcal{A})$ implies $x_e \in (\mathcal{G},\mathcal{A}), y_e \notin (\mathcal{G},\mathcal{A})$ and $y_e \in (\mathcal{H},\mathcal{A}), x_e \notin (\mathcal{H},\mathcal{A})$ this implies $x_e \notin (\mathcal{G},\mathcal{A})^c$, $y_e \in (\mathcal{G},\mathcal{A})^c$ and $y_e \notin (\mathcal{H},\mathcal{A})^c, x_e \in (\mathcal{H},\mathcal{A})^c$ or $x_e \in (\mathcal{H},\mathcal{A})^c, y_e \notin (\mathcal{H},\mathcal{A})^c$ and $y_e \in (\mathcal{G},\mathcal{A})^c, x_e \notin (\mathcal{G},\mathcal{A})^c$.

Let $(\mathcal{Q},\mathcal{A}) = (\mathcal{H},\mathcal{A})^c$ and $(\mathcal{F},\mathcal{A}) = (\mathcal{G},\mathcal{A})^c$, Then $(\mathcal{Q},\mathcal{A})$ and $(\mathcal{F},\mathcal{A})$ are soft $\alpha-$closed sets and $x_e \in (\mathcal{Q},\mathcal{A})$, $y_e \notin (\mathcal{Q},\mathcal{A})$ and $y_e \in (\mathcal{F},\mathcal{A})$, $x_e \notin (\mathcal{F},\mathcal{A})$. Also $(\mathcal{Q},\mathcal{A}) \cup (\mathcal{F},\mathcal{A}) = (\mathcal{G},\mathcal{A})^c \cup (\mathcal{H},\mathcal{A})^c = [(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})]^c = \phi^c = \mathcal{X}$. Conversely, let us suppose that for every two soft distinct points x_e and y_e of $\mathcal{X}$, there exists soft $\alpha-$closed sets $(\mathcal{Q},\mathcal{A})$ and $(\mathcal{F},\mathcal{A})$ such that $x_e \in (\mathcal{Q},\mathcal{A})$, $y_e \notin (\mathcal{Q},\mathcal{A})$ and $y_e \in (\mathcal{F},\mathcal{A})$, $x_e \notin (\mathcal{F},\mathcal{A})$ and $(\mathcal{Q},\mathcal{A}) \cup (\mathcal{F},\mathcal{A}) = \mathcal{X}$. We prove that $(\mathcal{X},\tau,\mathcal{A})$ be a soft Hausdorff space. We proceeds as fallow to catch the required space. $(\mathcal{G},\mathcal{A}) = (\mathcal{Q},\mathcal{A})^c$ and $(\mathcal{H},\mathcal{A}) = (\mathcal{F},\mathcal{A})^c$ then $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ are soft $\alpha-$open sets and $x_e \in (\mathcal{Q},\mathcal{A})$, $y_e \notin (\mathcal{Q},\mathcal{A})$ and $y_e \in (\mathcal{F},\mathcal{A})$, $x_e \notin (\mathcal{F},\mathcal{A})$. This implies, $y_e \notin (\mathcal{Q},\mathcal{A})$ and $x_e \notin (\mathcal{F},\mathcal{A})$ implies $y_e \in (\mathcal{Q},\mathcal{A})^c$ and $x_e \in (\mathcal{F},\mathcal{A})^c$ implies that $x_e \in (\mathcal{H},\mathcal{A})$ and $y_e \in (\mathcal{G},\mathcal{A})$. Moreover, $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = (\mathcal{F},\mathcal{A})^c \cap (\mathcal{Q},\mathcal{A})^c = [(\mathcal{F},\mathcal{A}) \cap (\mathcal{Q},\mathcal{A})]^c = \mathcal{X}^c = \phi$. This proves that for every two soft distinct points x_e and y_e of $\mathcal{X}$, there exists soft $\alpha-$open sets $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ such hat $x_e \in (\mathcal{G},\mathcal{A})$ and $y_e \in (\mathcal{H},\mathcal{A})$ such that $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = \phi$. This completes the proof. □

Theorem 4.18. *A soft topological space* $(\mathcal{X}, \tau, \mathcal{A})$ *is a* $\beta-$*Hausdorff space iff for every two distinct soft points* x_e *and* y_e *belong to* $\mathcal{X}$, *There exists soft* $\beta-$*closed sets* $(\mathcal{Q}, \mathcal{A})$ *and* $(\mathcal{F}, \mathcal{A})$ *such that* $x_e \in (\mathcal{Q}, \mathcal{A})$, $y_e \notin (\mathcal{Q}, \mathcal{A})$ *and* $y_e \in (\mathcal{F}, \mathcal{A})$, $x_e \notin (\mathcal{F}, \mathcal{A})$ *and* $(\mathcal{Q}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A}) = \mathcal{X}$.

Proof. Suppose $(\mathcal{X}, \tau, \mathcal{A})$ be a soft $\beta-$Hausdorff space. Then according to definition for $x_e, y_e \in \mathcal{X}$ such that $x_e \neq y_e$. If we can find two soft $\beta-$open sets such that $x_e \in (\mathcal{G}, \mathcal{A})$ and $y_e \in (\mathcal{H}, \mathcal{A})$ moreover $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi$. Since $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ are mutually exclusive. So $x_e \in (\mathcal{G}, \mathcal{A})$ and $y_e \in (\mathcal{H}, \mathcal{A})$ implies $x_e \in (\mathcal{G}, \mathcal{A}), y_e \notin (\mathcal{G}, \mathcal{A})$ and $y_e \in (\mathcal{H}, \mathcal{A}), x_e \notin (\mathcal{H}, \mathcal{A})$ this implies $x_e \notin (\mathcal{G}, \mathcal{A})^c$, $y_e \in (\mathcal{G}, \mathcal{A})^c$ and $y_e \notin (\mathcal{H}, \mathcal{A})^c, x_e \in (\mathcal{H}, \mathcal{A})^c$ or $x_e \in (\mathcal{H}, \mathcal{A})^c, y_e \notin (\mathcal{H}, \mathcal{A})^c$ and $y_e \in (\mathcal{G}, \mathcal{A})^c, x_e \notin (\mathcal{G}, \mathcal{A})^c$.

Let $(\mathcal{Q}, \mathcal{A}) = (\mathcal{H}, \mathcal{A})^c$ and $(\mathcal{F}, \mathcal{A}) = (\mathcal{G}, \mathcal{A})^c$, Then $(\mathcal{Q}, \mathcal{A})$ and $(\mathcal{F}, \mathcal{A})$ are soft $\beta-$closed sets and $x_e \in (\mathcal{Q}, \mathcal{A})$, $y_e \notin (\mathcal{Q}, \mathcal{A})$ and $y_e \in (\mathcal{F}, \mathcal{A})$, $x_e \notin (\mathcal{F}, \mathcal{A})$. Also $(\mathcal{Q}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A}) = (\mathcal{G}, \mathcal{A})^c \cup (\mathcal{H}, \mathcal{A})^c = [(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A})]^c = \phi^c = \mathcal{X}$. Conversely, let us suppose that for every two soft distinct points x_e and y_e of $\mathcal{X}$, there exists soft $\beta-$closed sets $(\mathcal{Q}, \mathcal{A})$ and $(\mathcal{F}, \mathcal{A})$ such that $x_e \in (\mathcal{Q}, \mathcal{A})$, $y_e \notin (\mathcal{Q}, \mathcal{A})$ and $y_e \in (\mathcal{F}, \mathcal{A})$, $x_e \notin (\mathcal{F}, \mathcal{A})$ and $(\mathcal{Q}, \mathcal{A}) \cup (\mathcal{F}, \mathcal{A}) = \mathcal{X}$. We prove that $(\mathcal{X}, \tau, \mathcal{A})$ be a soft Hausdorff space. We proceeds as fallow to catch the required space. $(\mathcal{G}, \mathcal{A}) = (\mathcal{Q}, \mathcal{A})^c$ and $(\mathcal{H}, \mathcal{A}) = (\mathcal{F}, \mathcal{A})^c$ then $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ are soft $\beta-$open sets and $x_e \in (\mathcal{Q}, \mathcal{A})$, $y_e \notin (\mathcal{Q}, \mathcal{A})$ and $y_e \in (\mathcal{F}, \mathcal{A})$, $x_e \notin (\mathcal{F}, \mathcal{A})$. This implies, $y_e \notin (\mathcal{Q}, \mathcal{A})$ and $x_e \notin (\mathcal{F}, \mathcal{A})$ implies $y_e \in (\mathcal{Q}, \mathcal{A})^c$ and $x_e \in (\mathcal{F}, \mathcal{A})^c$ implies that $x_e \in (\mathcal{H}, \mathcal{A})$ and $y_e \in (\mathcal{G}, \mathcal{A})$. Moreover, $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = (\mathcal{F}, \mathcal{A})^c \cap (\mathcal{Q}, \mathcal{A})^c = [(\mathcal{F}, \mathcal{A}) \cap (\mathcal{Q}, \mathcal{A})]^c = \mathcal{X}^c = \phi$. This proves that for every two soft distinct points x_e and y_e of $\mathcal{X}$, there exists soft $\beta-$open sets $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ such hat $x_e \in (\mathcal{G}, \mathcal{A})$ and $y_e \in (\mathcal{H}, \mathcal{A})$ such that $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi$. This completes the proof. □

Theorem 4.19. *If* $\mathcal{X}$ *and* $\mathcal{Y}$ *are two soft Hausdorff spaces then Their Product is also soft Hausdorff space.*

Proof. Let $(\mathcal{C}, \mathcal{A})$ and $(\mathcal{D}, \mathcal{A})$ be two soft points such that $(\mathcal{C}, \mathcal{A}) = (x_{e_1}, x_{e_2})$ and $(\mathcal{D}, \mathcal{A}) = (y_{e_1}, y_{e_2})$ and moreover they distinct. Suppose $(x_{e_1}, y_{e_1}) \in \mathcal{X}$ and $(x_{e_2}, y_{e_2}) \in \mathcal{Y}$ if $(x_{e_1} \neq y_{e_2})$ are two distinct points of $\mathcal{X}$ but $\mathcal{X}$ is given to be soft T_2 space so according to definition we can find two distinct soft open sets $(\mathcal{G}_1, \mathcal{A})$ and $(\mathcal{H}_1, \mathcal{A})$ such that $x_{e_1} \in (\mathcal{G}_1, \mathcal{A})$ and $y_{e_1} \in (\mathcal{H}_1, \mathcal{A})$.

Now $(\mathcal{G}_1, \mathcal{A}) \times \mathcal{Y}$ and $(\mathcal{H}_1, \mathcal{A}) \times \mathcal{Y}$ are two distinct soft open sets in $\mathcal{X} \times \mathcal{Y}$ containing $(\mathcal{C}, \mathcal{A})$ and $(\mathcal{D}, \mathcal{A})$ respectively that is $(\mathcal{C}, \mathcal{A}) \in (\mathcal{G}_1, \mathcal{A}) \times \mathcal{Y}$ and $(\mathcal{D}, \mathcal{A}) \in (\mathcal{H}_1, \mathcal{A}) \times \mathcal{Y}$ and $((\mathcal{G}_1, \mathcal{A}) \times \mathcal{Y}) \cap ((\mathcal{H}_1, \mathcal{A}) \times \mathcal{Y}) = \phi$.

Thus $\mathcal{X} \times \mathcal{Y}$ is a soft T_2 space. Now if $(x_{e_2} \neq (y_{e_2})$ are two distinct points of $\mathcal{X}$ but $\mathcal{Y}$ is given to be soft T_2 space so according to definition we can find two distinct soft open sets $(\mathcal{G}_2, \mathcal{A})$ and $(\mathcal{H}_2, \mathcal{A})$ such that $((x_{e_2}) \in (\mathcal{G}_2, \mathcal{A}))$ and $((y_{e_2}) \in (\mathcal{H}_2, \mathcal{A}))$. Now $\mathcal{X} \times (\mathcal{G}_2, \mathcal{A})$ and $\mathcal{X} \times (\mathcal{H}_2, \mathcal{A})$ are two distinct soft open sets in $\mathcal{X} \times \mathcal{Y}$ containing $(\mathcal{C}, \mathcal{A})$ and $(\mathcal{D}, \mathcal{A})$ respectively that is $(\mathcal{C}, \mathcal{A}) \in \mathcal{X} \times (\mathcal{G}_2, \mathcal{A})$ and $(\mathcal{D}, \mathcal{A}) \in \mathcal{X} \times (\mathcal{H}_2, \mathcal{A})$ such that $(\mathcal{X} \times (\mathcal{G}_2, \mathcal{A})) \cap (\mathcal{X} \times (\mathcal{H}_2, \mathcal{A})) = \phi$. Thus for both the cases we proved the result. Hence the product of any finite number of soft Hausdorff spaces is a soft Hausdorff space. □

Theorem 4.20. *If $\mathcal{X}$ and $\mathcal{Y}$ are two soft $\alpha-$Hausdorff spaces then Their Product is also soft $\alpha-$Hausdorff space.*

Proof. Let $(\mathcal{C}, \mathcal{A})$ and $(\mathcal{D}, \mathcal{A})$ be two soft points such that $(\mathcal{C}, \mathcal{A}) = (x_{e_1}, x_{e_2})$ and $(\mathcal{D}, \mathcal{A}) = (y_{e_1}, y_{e_2})$ and moreover they distinct. Suppose $(x_{e_1}, y_{e_1}) \in \mathcal{X}$ and $(x_{e_2}, y_{e_2}) \in \mathcal{Y}$ if $(x_{e_1} \neq y_{e_2})$ are two distinct points of $\mathcal{X}$ but $\mathcal{X}$ is given to be soft T_2 space so according to definition we can find two distinct soft $\alpha-$open sets $(\mathcal{G}_1, \mathcal{A})$ and $(\mathcal{H}_1, \mathcal{A})$ such that $x_{e_1} \in (\mathcal{G}_1, \mathcal{A})$ and $y_{e_1} \in (\mathcal{H}_1, \mathcal{A})$. Now $(\mathcal{G}_1, \mathcal{A}) \times \mathcal{Y}$ and $(\mathcal{H}_1, \mathcal{A}) \times \mathcal{Y}$ are two distinct soft $\alpha-$open sets in $\mathcal{X} \times \mathcal{Y}$ containing $(\mathcal{C}, \mathcal{A})$ and $(\mathcal{D}, \mathcal{A})$ respectively that is $(\mathcal{C}, \mathcal{A}) \in (\mathcal{G}_1, \mathcal{A}) \times \mathcal{Y}$ and $(\mathcal{D}, \mathcal{A}) \in (\mathcal{H}_1, \mathcal{A}) \times \mathcal{Y}$ and $((\mathcal{G}_1, \mathcal{A}) \times \mathcal{Y}) \cap ((\mathcal{H}_1, \mathcal{A}) \times \mathcal{Y}) = \phi$.

Thus $\mathcal{X} \times \mathcal{Y}$ is a soft T_2 space. Now if $(x_{e_2} \neq (y_{e_2})$ are two distinct points of $\mathcal{X}$ but $\mathcal{Y}$ is given to be soft $\alpha - T_2$ space so according to definition we can find two distinct soft $\alpha-$open sets $(\mathcal{G}_2, \mathcal{A})$ and $(\mathcal{H}_2, \mathcal{A})$ such that $((x_{e_2}) \in (\mathcal{G}_2, \mathcal{A}))$ and $((y_{e_2}) \in (\mathcal{H}_2, \mathcal{A}))$. Now $\mathcal{X} \times (\mathcal{G}_2, \mathcal{A})$ and $\mathcal{X} \times (\mathcal{H}_2, \mathcal{A})$ are two distinct soft $\alpha-$open sets in $\mathcal{X} \times \mathcal{Y}$ containing $(\mathcal{C}, \mathcal{A})$ and $(\mathcal{D}, \mathcal{A})$ respectively that is $(\mathcal{C}, \mathcal{A}) \in \mathcal{X} \times (\mathcal{G}_2, \mathcal{A})$ and $(\mathcal{D}, \mathcal{A}) \in \mathcal{X} \times (\mathcal{H}_2, \mathcal{A})$ such that $(\mathcal{X} \times (\mathcal{G}_2, \mathcal{A})) \cap (\mathcal{X} \times (\mathcal{H}_2, \mathcal{A})) = \phi$. Thus for both the cases we proved the result. Hence the product of any finite number of soft $\alpha-$Hausdorff spaces is a soft $\alpha-$Hausdorff space. □

Theorem 4.21. *If $\mathcal{X}$ and $\mathcal{Y}$ are two soft $\beta-$Hausdorff spaces then Their Product is also soft $\beta-$Hausdorff space.*

Proof. Let $(\mathcal{C},\mathcal{A})$ and $(\mathcal{D},\mathcal{A})$ be two soft points such that $(\mathcal{C},\mathcal{A}) = (x_{e_1}, x_{e_2})$ and $(\mathcal{D},\mathcal{A}) = (y_{e_1}, y_{e_2})$ and moreover they distinct. Suppose $(x_{e_1}, y_{e_1}) \in \mathcal{X}$ and $(x_{e_2}, y_{e_2}) \in \mathcal{Y}$ if $(x_{e_1} \neq y_{e_2})$ are two distinct points of $\mathcal{X}$ but $\mathcal{X}$ is given to be soft T_2 space so according to definition we can find two distinct soft semi-open sets $(\mathcal{G}_1,\mathcal{A})$ and $(\mathcal{H}_1,\mathcal{A})$ such that $x_{e_1} \in (\mathcal{G}_1,\mathcal{A})$ and $y_{e_1} \in (\mathcal{H}_1,\mathcal{A})$. Now $(\mathcal{G}_1,\mathcal{A}) \times \mathcal{Y}$ and $(\mathcal{H}_1,\mathcal{A}) \times \mathcal{Y}$ are two distinct soft $\beta-$open sets in $\mathcal{X} \times \mathcal{Y}$ containing $(\mathcal{C},\mathcal{A})$ and $(\mathcal{D},\mathcal{A})$ respectively that is $(\mathcal{C},\mathcal{A}) \in (\mathcal{G}_1,\mathcal{A}) \times \mathcal{Y}$ and $(\mathcal{D},\mathcal{A}) \in (\mathcal{H}_1,\mathcal{A}) \times \mathcal{Y}$ and $((\mathcal{G}_1,\mathcal{A}) \times \mathcal{Y}) \cap ((\mathcal{H}_1,\mathcal{A}) \times \mathcal{Y}) = \phi$.

Thus $\mathcal{X}\times\mathcal{Y}$ is a soft T_2 space. Now if $(x_{e_2} \neq (y_{e_2})$ are two distinct points of $\mathcal{X}$ but $\mathcal{Y}$ is given to be soft $\beta - T_2$ space so according to definition we can find two distinct soft $\beta-$open sets $(\mathcal{G}_2,\mathcal{A})$ and $(\mathcal{H}_2,\mathcal{A})$ such that $((x_{e_2}) \in (\mathcal{G}_2,\mathcal{A}))$ and $((y_{e_2}) \in (\mathcal{H}_2,\mathcal{A}))$. Now $\mathcal{X}\times(\mathcal{G}_2,\mathcal{A})$ and $\mathcal{X}\times(\mathcal{H}_2,\mathcal{A})$ are two distinct soft $\beta-$open sets in $\mathcal{X}\times\mathcal{Y}$ containing $(\mathcal{C},\mathcal{A})$ and $(\mathcal{D},\mathcal{A})$ respectively that is $(\mathcal{C},\mathcal{A}) \in \mathcal{X} \times (\mathcal{G}_2,\mathcal{A})$ and $(\mathcal{D},\mathcal{A}) \in \mathcal{X} \times (\mathcal{H}_2,\mathcal{A})$ such that $(\mathcal{X}\times(\mathcal{G}_2,\mathcal{A}))\cap(\mathcal{X}\times(\mathcal{H}_2,\mathcal{A})) = \phi$. Thus for both the cases we proved the result. Hence the product of any finite number of soft $\beta-$Hausdorff spaces is a soft $\beta-$Hausdorff space. □

A natural question arises about the product of any number of soft Hausdorff space. To get positive reply we proceeds as fallows.

Theorem 4.22. *The product of any number of soft Hausdorff spaces is a soft Hausdorff space.*

Proof. To fix the idea let us suppose that $\mathcal{X} = \prod_{i\in I}\mathcal{X}_i$ provided $\mathcal{X}_i\ \forall\ i \in I$ is a soft Hausdorff space at its own right. We need to show that $\mathcal{X}$ is soft Hausdorff space. For this let x_e and y_e be two soft arbitrary points of $\prod_{i\in I}\mathcal{X}_i$ such that they are disjoint. This implies $x_e = (x_{e_i})_{i\in I}$ and $y_e = (y_{e_i})_{i\in I}$ where x_{e_i} and $y_{e_i} \in \mathcal{X}_i\ \forall\ i \in I$. But x_e and y_e are distinct, so automatically $(x_{e_i})_{i\in I}$ and $(y_{e_i})_{i\in I}$ are distinct which leads to x_{e_i} and y_{e_i} are distinct for some $i \in I$.

Since the soft projection $\pi_i : \prod_{i\in I}\ \mathcal{X}_i \to \mathcal{X}_i$ defined by $\pi_1\ (x_e) = x_{e_i}$ for all $x_{e_i} = (x_{e_i})_{i\in I}$. Since x_{e_i} and $y_{e_i} \in X_i$ such that x_{e_i} and y_{e_i} are distinct and more over X_i is soft Hausdorff space. So definitely the characteristics of Hausdorff space will come in use so according to definition there must exist soft open sets $(\mathcal{G}_i,\mathcal{A})$ and $(\mathcal{H}_i,\mathcal{A})$ in $\mathcal{X}_i$ such that $x_{e_1} \in (\mathcal{G}_1,\mathcal{A})$, $y_{e_1} \in (\mathcal{H}_1,\mathcal{A})$, $(\mathcal{G}_1,\mathcal{A}) \cap (\mathcal{H}_1,\mathcal{A}) = \phi$

$x_{e_2} \in (\mathcal{G}_2,\mathcal{A})$, $y_{e_2} \in (\mathcal{H}_2,\mathcal{A})$, $(\mathcal{G}_2,\mathcal{A}) \cap (\mathcal{H}_2,\mathcal{A}) = \phi$ $x_{e_3} \in (\mathcal{G}_3,\mathcal{A})$, $y_{e_3} \in (\mathcal{H}_3,\mathcal{A})$, $(\mathcal{G}_3,\mathcal{A}) \cap (\mathcal{H}_3,\mathcal{A}) = \phi$ and in general $x_{e_i} \in (\mathcal{G}_i,\mathcal{A})$, $y_{e_i} \in (\mathcal{H}_i,\mathcal{A})$ and $(\mathcal{G}_i,\mathcal{A}) \cap (\mathcal{H}_i,\mathcal{A}) = \phi$. This implies $\pi_i\ (x) \in (\mathcal{G}_i,\mathcal{A})$ and $\pi_i\ (y) \in (\mathcal{H}_i,\mathcal{A})$ this implies $(x_e) \in {\pi_i}^{-1}(\mathcal{G}_i,\mathcal{A})$ and $(y_e) \in {\pi_i}^{-1}(\mathcal{H}_i,\mathcal{A})$. Let $(\mathcal{G}_2,\mathcal{A}) = {\pi_i}^{-1}(\mathcal{G}_i,\mathcal{A})$, and $(\mathcal{H},\mathcal{A}) = {\pi_i}^{-1}(\mathcal{H}_i,\mathcal{A})$. Then obviously, $(x_e) \in (\mathcal{G}_2,\mathcal{A})$ and$(y_e) \in (\mathcal{H}_1,\mathcal{A})$. Since ${\pi_i}^{-1}(\mathcal{G}_i,\mathcal{A})$ and ${\pi_i}^{-1}(\mathcal{H}_i,\mathcal{A})$ are soft open in $\prod_{i\in I}\mathcal{X}_i$, so do $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$. So to attain the required result it is enough to show that $(\mathcal{G}_2,\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = \phi$. Consider $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = {\pi_i}^{-1}(\mathcal{G}_i,\mathcal{A}) \cap {\pi_i}^{-1}(\mathcal{H}_i,\mathcal{A}) = {\pi_i}^{-1}[(\mathcal{G}_2,\mathcal{A}) \cap (\mathcal{H}_i,\mathcal{A})] = {\pi_i}^{-1}(\phi) = \phi$. This completes the proof. ${\pi_i}^{-1}[(\mathcal{G}_2,\mathcal{A}) \cap (\mathcal{H}_i,\mathcal{A})]$. □

Theorem 4.23. *Suppose $(\mathcal{X},\tau,\mathcal{A})$ be a soft $\alpha-$Hausdorff space, then the following statements are equivalent*
(1) $(\mathcal{X},\tau,\mathcal{A})$ is soft $\alpha-$Hausdorff space.
(2) The diagonal $\Delta = \{(x_e,x_e) : x_e \in \mathcal{X}\}$ is soft $\alpha-$closed in $\mathcal{X}\times\mathcal{X}$.

Proof. Suppose (1) holds. We have to prove that the diagonal $\Delta = \{(x_e,x_e) : x_e \in \mathcal{X}\}$ is soft $\alpha-$closed in $\mathcal{X}\times\mathcal{X}$. Equivalently we will prove $\Delta^c = \{(x_e,y_e)\}$ and $x_e \neq y_e$ is soft $\alpha-$open in $\mathcal{X}\times\mathcal{X}$. Suppose $(x_e,y_e) \in \Delta^c$, Then $x_e,y_e \in \mathcal{X}$ such that $x_e > y_e$ or $x_e < y_e$ Since $(\mathcal{X},\tau,\mathcal{A})$ be a soft $\alpha-$Hausdorff space, so according to definition, there exist soft $\alpha-$open set $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ such that $x_e \in (\mathcal{G},\mathcal{A})$, $y_e \in (\mathcal{H},\mathcal{A})$ and $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = \phi$. This implies $(x_e,y_e) \in (\mathcal{G},\mathcal{A}) \times (\mathcal{H},\mathcal{A})$. Since $x_e \neq y_e$, so $(x_e,y_e) \in \Delta^c$. Therefore, $(\mathcal{G},\mathcal{A}) \times (\mathcal{H},\mathcal{A}) \subseteq \Delta^c$. Also since $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ are soft $\alpha-$open in $\mathcal{X}$ so $(\mathcal{G},\mathcal{A}) \times (\mathcal{H},\mathcal{A})$ is $\alpha-$soft open in the soft product spacee $\mathcal{X}\times\mathcal{X}$. so $(x_e,y_e) \in (\mathcal{G},\mathcal{A}) \times (\mathcal{H},\mathcal{A}) \subseteq \Delta^c$. This implies (x_e,y_e) is soft interior point of Δ^c. But (x_e,y_e) is soft temporary point of Δ^c. So every soft point of Δ^c is soft interior point of Δ^c. result in Δ^c is soft $\alpha-$open in $\mathcal{X}\times\mathcal{X}$. Suppose (2) holds. We have to prove the second for this suppose $x_e,y_e \in \mathcal{X}$ such that $x_e > y_e$ or $x_e < y_e$. Since $x_e > y_e$ or $x_e < y_e$ this means that the given points are distinct so $(x_e,y_e) \in \Delta^c$. But Δ^c is soft $\alpha-$open in $\mathcal{X}\times\mathcal{X}$. So $(x,y)\Delta^c$ is an interior point of Δ^c. So by definition there exists soft $\alpha-$open sets $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ such that so $(\mathcal{G},\mathcal{A}) \times (\mathcal{H},\mathcal{A}) \subseteq \Delta^c$ implies that $x_e \in (\mathcal{G},\mathcal{A})$, $y_e \in (\mathcal{H},\mathcal{A})$. Since x_e and y are arbitrary pints of $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ such that $x_e > y_e$ or $x_e < y_e$. So $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ contains no common point of, so $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = \phi$. Thus there exist soft $\alpha-$open sets $(\mathcal{G},\mathcal{A})$ and $(\mathcal{H},\mathcal{A})$ such that $x_e \in (\mathcal{G},\mathcal{A})$, $y_e \in (\mathcal{H},\mathcal{A})$ and $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A}) = \phi$. Thus

$(\mathcal{X}, \tau, \mathcal{A})$ be a soft $\alpha-$Hausdorff space. This finishes the proof. □

Theorem 4.24. *Suppose $(\mathcal{X}, \tau, \mathcal{A})$ be a soft $\beta-$Hausdorff space, then the following statements are equivalent*
(1) $(\mathcal{X}, \tau, \mathcal{A})$ is soft $\beta-$Hausdorff space.
(2) The diagonal $\Delta = \{(x_e, x_e) : x_e \in \mathcal{X}\}$ is soft $\beta-$closed in $\mathcal{X} \times \mathcal{X}$.

Proof. Suppose (1) holds. We have to prove that the diagonal $\Delta = \{(x_e, x_e) : x_e \in \mathcal{X}\}$ is soft $\beta-$closed in $\mathcal{X} \times \mathcal{X}$. Equivalently we will prove $\Delta^c = \{(x_e, y_e)\}$ and $x_e \neq y_e$ is soft $\beta-$open in $\mathcal{X} \times \mathcal{X}$. Suppose $(x_e, y_e) \in \Delta^c$, Then $x_e, y_e \in \mathcal{X}$ such that $x_e > y_e$ or $x_e < y_e$ Since $(\mathcal{X}, \tau, \mathcal{A})$ be a soft $\beta-$Hausdorff space, so according to definition, there exist soft $\beta-$open set $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ such that $x_e \in (\mathcal{G}, \mathcal{A})$, $y_e \in (\mathcal{H}, \mathcal{A})$ and $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi$. This implies $(x_e, y_e) \in (\mathcal{G}, \mathcal{A}) \times (\mathcal{H}, \mathcal{A})$. Since $x_e \neq y_e$, so $(x_e, y_e) \in \Delta^c$. Therefore, $(\mathcal{G}, \mathcal{A}) \times (\mathcal{H}, \mathcal{A}) \subseteq \Delta^c$. Also since $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ are soft $\beta-$open in $\mathcal{X}$ so $(\mathcal{G}, \mathcal{A}) \times (\mathcal{H}, \mathcal{A})$ is $\beta-$soft open in the soft product spacee $\mathcal{X} \times \mathcal{X}$. so $(x_e, y_e) \in (\mathcal{G}, \mathcal{A}) \times (\mathcal{H}, \mathcal{A}) \subseteq \Delta^c$. This implies (x_e, y_e) is soft interior point of Δ^c. But (x_e, y_e) is soft temporary point of Δ^c. So every soft point of Δ^c is soft interior point of Δ^c. result in Δ^c is soft $\beta-$open in $\mathcal{X} \times \mathcal{X}$. Suppose (2) holds. We have to prove the second for this suppose $x_e, y_e \in \mathcal{X}$ such that $x_e > y_e$ or $x_e < y_e$. Since $x_e > y_e$ or $x_e < y_e$ this means that the given points are distinct so $(x_e, y_e) \in \Delta^c$. But Δ^c is soft $\beta-$open in $\mathcal{X} \times \mathcal{X}$. So $(x, y)\Delta^c$ is an interior point of Δ^c. So by definition there exists soft $\beta-$open sets $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ such that so $(\mathcal{G}, \mathcal{A}) \times (\mathcal{H}, \mathcal{A}) \subseteq \Delta^c$ implies that $x_e \in (\mathcal{G}, \mathcal{A})$, $y_e \in (\mathcal{H}, \mathcal{A})$. Since x_e and y are arbitrary pints of $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ such that $x_e > y_e$ or $x_e < y_e$. So $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ contains no common point of, so $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi$. Thus there exist soft $\beta-$open sets $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ such that $x_e \in (\mathcal{G}, \mathcal{A})$, $y_e \in (\mathcal{H}, \mathcal{A})$ and $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi$. Thus $(\mathcal{X}, \tau, \mathcal{A})$ be a soft $\beta-$Hausdorff space. This finishes the proof. □

Theorem 4.25. *Let $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$ be a soft bi topological space. Suppose $(\mathcal{X}, \tau_1, \tau_2, \mathcal{A})$ is soft T_1 space. Then a point $x_e \in X$ is a soft limit point of $(\mathcal{F}, \mathcal{A})$ iff every soft open set containing x_e contains infinitely many points of $(\mathcal{F}, \mathcal{A})$.*

Proof. Suppose $x_e\mathcal{X}$ is a soft limit point of $(\mathcal{F}, \mathcal{A})$. We are to prove very soft open set containing x contains infinitely many points of $(\mathcal{F}, \mathcal{A})$. We contra positively suppose that there is soft open set $(\mathcal{G}, \mathcal{A})$. having x

such that $(\mathcal{G},\mathcal{A})$ contains only finite number of points of $(\mathcal{F},\mathcal{A})$. that is $(\mathcal{G},\mathcal{A}) \cap (\mathcal{G},\mathcal{A})$ is finite implies that $[(\mathcal{G},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A})$ is finite. Suppose $[(\mathcal{G},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A}) = (\mathcal{H},\mathcal{A})$. Then $(\mathcal{H},\mathcal{A})$ is definitely finite and $(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = (\mathcal{H},\mathcal{A}) \cup \{x_e\}$ if $x_e \in (\mathcal{F},\mathcal{A})$ and $(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = (\mathcal{H},\mathcal{A})$ if $x_e \notin (\mathcal{F},\mathcal{A})$.

Now $(\mathcal{H},\mathcal{A})$ being a finite soft sub set of soft T_1 space is soft closed $(\mathcal{H},\mathcal{A})^c$ is soft open. Let $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})^c = (\mathcal{L},\mathcal{A})$. Then $(\mathcal{L},\mathcal{A})$ is soft open as $(\mathcal{G},\mathcal{A})$ and $(\mathcal{G},\mathcal{A})^c$ are soft open sets and $x_e \in (\mathcal{H},\mathcal{A})$. If $x_e \in (\mathcal{H},\mathcal{A})$. Then $(\mathcal{L},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = [(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})^c] \cap ((\mathcal{F},\mathcal{A}) = \{x_e\}$. If $x_e \notin (\mathcal{F},\mathcal{A})$. $(\mathcal{L},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = [(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})^c] \cap ((\mathcal{F},\mathcal{A}) = \phi$. Thus in both the cases $[(\mathcal{L},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A}) = \phi$.

This implies x_e is not soft limit point of $(\mathcal{F},\mathcal{A})$ but this is openly a contradiction to the hypothesis that x_e is soft limit point of $(\mathcal{F},\mathcal{A})$. This contradiction is taking birth due to our wrong supposition. So every soft open set containing x_e contains infinitely many points of $(\mathcal{F},\mathcal{A})$. Conversely suppose that every soft open set containing x_e contains infinitely many points of $(\mathcal{F},\mathcal{A})$. We are to prove that x is soft limit point of $(\mathcal{F},\mathcal{A})$. For this let $(\mathcal{G},\mathcal{A})$. be arbitrary soft open set containing x_e, then definitely $(\mathcal{G},\mathcal{A})$. contain infinitely many points of $(\mathcal{F},\mathcal{A})$. other than x_e that is $[(\mathcal{G},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A}) \neq \phi$. this implies x_e is soft limit point of $(\mathcal{F},\mathcal{A})$. □

Theorem 4.26. *Let $(\mathcal{X},\tau_1,\tau_2,\mathcal{A})$ be a soft bi topological space. Suppose $(\mathcal{X},\tau_1,\tau_2,\mathcal{A})$ is soft βT_1 space. Then a point $x_e \in X$ is a soft limit point of $(\mathcal{F},\mathcal{A})$ iff every soft $\beta-$open set containing x_e contains infinitely many points of $(\mathcal{F},\mathcal{A})$.*

Proof. Suppose $x_e\mathcal{X}$ is a soft limit point of $(\mathcal{F},\mathcal{A})$. We are to prove very soft $\beta-$open set containing x contains infinitely many points of $(\mathcal{F},\mathcal{A})$. We contra positively suppose that there is soft $\beta-$open set $(\mathcal{G},\mathcal{A})$. having x such that $(\mathcal{G},\mathcal{A})$ contains only finite number of points of $(\mathcal{F},\mathcal{A})$. that is $(\mathcal{G},\mathcal{A}) \cap (\mathcal{G},\mathcal{A})$ is finite implies that $[(\mathcal{G},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A})$ is finite. Suppose $[(\mathcal{G},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A}) = (\mathcal{H},\mathcal{A})$. Then $(\mathcal{H},\mathcal{A})$ is definitely finite and $(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = (\mathcal{H},\mathcal{A}) \cup \{x_e\}$ if $x_e \in (\mathcal{F},\mathcal{A})$ and $(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = (\mathcal{H},\mathcal{A})$ if $x_e \notin (\mathcal{F},\mathcal{A})$.

Now $(\mathcal{H},\mathcal{A})$ being a finite soft sub set of soft $\beta-T_1$ space is soft $\beta-$closed $(\mathcal{H},\mathcal{A})^c$ is soft $\beta-$open. Let $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})^c = (\mathcal{L},\mathcal{A})$. Then $(\mathcal{L},\mathcal{A})$ is soft open as $(\mathcal{G},\mathcal{A})$ and $(\mathcal{G},\mathcal{A})^c$ are soft $\beta-$open sets and $x_e \in (\mathcal{H},\mathcal{A})$. If $x_e \in (\mathcal{H},\mathcal{A})$. Then $(\mathcal{L},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = [(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})^c] \cap ((\mathcal{F},\mathcal{A}) = \{x_e\}$. If $x_e \notin (\mathcal{F},\mathcal{A})$. $(\mathcal{L},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = [(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})^c] \cap ((\mathcal{F},\mathcal{A}) = \phi$. Thus in both the cases $[(\mathcal{L},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A}) = \phi$.

This implies x_e is not soft limit point of $(\mathcal{F},\mathcal{A})$ but this is openly a contradiction to the hypothesis that x_e is soft limit point of $(\mathcal{F},\mathcal{A})$. This contradiction is taking birth due to our wrong supposition. So every soft $\beta-$open set containing x_e contains infinitely many points of $(\mathcal{F},\mathcal{A})$. Conversely suppose that every soft $\beta-$open set containing x_e contains infinitely many points of $(\mathcal{F},\mathcal{A})$. We are to prove that x is soft limit point of $(\mathcal{F},\mathcal{A})$. For this let $(\mathcal{G},\mathcal{A})$. be arbitrary soft $\beta-$open set containing x_e, then definitely $(\mathcal{G},\mathcal{A})$. contain infinitely many points of $(\mathcal{F},\mathcal{A})$. other than x_e that is $[(\mathcal{G},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A}) \neq \phi$. this implies x_e is soft limit point of $(\mathcal{F},\mathcal{A})$. □

Theorem 4.27. *Let $(\mathcal{X},\tau_1,\tau_2,\mathcal{A})$ be a soft bi topological space. Suppose $(\mathcal{X},\tau_1,\tau_2,\mathcal{A})$ is soft $P-T_1$ space. Then a point $x_e \in X$ is a soft limit point of $(\mathcal{F},\mathcal{A})$ iff every soft $P-$open set containing x_e contains infinitely many points of $(\mathcal{F},\mathcal{A})$.*

Proof. Suppose $x_e\mathcal{X}$ is a soft limit point of $(\mathcal{F},\mathcal{A})$. We are to prove very soft $P-$open set containing x contains infinitely many points of $(\mathcal{F},\mathcal{A})$. We contra positively suppose that there is soft $P-$open set $(\mathcal{G},\mathcal{A})$. having x such that $(\mathcal{G},\mathcal{A})$ contains only finite number of points of $(\mathcal{F},\mathcal{A})$. that is $(\mathcal{G},\mathcal{A}) \cap (\mathcal{G},\mathcal{A})$ is finite implies that $[(\mathcal{G},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A})$ is finite. Suppose $[(\mathcal{G},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A}) = (\mathcal{H},\mathcal{A})$. Then $(\mathcal{H},\mathcal{A})$ is definitely finite and $(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = (\mathcal{H},\mathcal{A}) \cup \{x_e\}$ if $x_e \in (\mathcal{F},\mathcal{A})$ and $(\mathcal{G},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = (\mathcal{H},\mathcal{A})$ if $x_e \notin (\mathcal{F},\mathcal{A})$.

Now $(\mathcal{H},\mathcal{A})$ being a finite soft sub set of soft $P-T_1$ space is soft $P-$closed $(\mathcal{H},\mathcal{A})^c$ is soft $P-$open. Let $(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})^c = (\mathcal{L},\mathcal{A})$. Then $(\mathcal{L},\mathcal{A})$ is soft open as $(\mathcal{G},\mathcal{A})$ and $(\mathcal{G},\mathcal{A})^c$ are soft $P-$open sets and $x_e \in (\mathcal{H},\mathcal{A})$. If $x_e \in (\mathcal{H},\mathcal{A})$. Then $(\mathcal{L},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = [(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})^c] \cap ((\mathcal{F},\mathcal{A}) = \{x_e\}$. If $x_e \notin (\mathcal{F},\mathcal{A})$. $(\mathcal{L},\mathcal{A}) \cap (\mathcal{F},\mathcal{A}) = [(\mathcal{G},\mathcal{A}) \cap (\mathcal{H},\mathcal{A})^c] \cap ((\mathcal{F},\mathcal{A}) = \phi$. Thus in both the cases $[(\mathcal{L},\mathcal{A}) - \{x_e\}] \cap (\mathcal{F},\mathcal{A}) = \phi$.

This implies x_e is not soft limit point of $(\mathcal{F},\mathcal{A})$ but this is openly a contradiction to the hypothesis that x_e is soft limit point of $(\mathcal{F},\mathcal{A})$. This contradiction is taking birth due to our wrong supposition. So every soft

$P-$open set containing x_e contains infinitely many points of $(\mathcal{F}, \mathcal{A})$. Conversely suppose that every soft $P-$open set containing x_e contains infinitely many points of $(\mathcal{F}, \mathcal{A})$. We are to prove that x is soft limit point of $(\mathcal{F}, \mathcal{A})$. For this let $(\mathcal{G}, \mathcal{A})$. be arbitrary soft $P-$open set containing x_e, then definitely $(\mathcal{G}, \mathcal{A})$. contain infinitely many points of $(\mathcal{F}, \mathcal{A})$. other than x_e that is $[(\mathcal{G}, \mathcal{A}) - \{x_e\}] \cap (\mathcal{F}, \mathcal{A}) \neq \phi$. this implies x_e is soft limit point of $(\mathcal{F}, \mathcal{A})$. □

Theorem 4.28. *In a soft $\alpha-$Hausdorff space every soft sequence converges to at most one point.*

Proof. Suppose $(\mathcal{X}, \tau, \mathcal{A})$ be a soft $\alpha-$Hausdorff space and $\{x_n\}$ be a soft sequence approaches the soft limit l. We prove this result by contradiction. Suppose the limit of the soft sequence is not unique and converges to another soft real number say m which is entirely different of l. Since $(\mathcal{X}, \tau, \mathcal{A})$ be a soft $\alpha-$Hausdorff space. So by definition we can find two soft mutually exclusive $\alpha-$open sets $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ such that $l \in (\mathcal{G}, \mathcal{A})$ and $m \in (\mathcal{H}, \mathcal{A})$ such that $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi$. Since x_n converges to l so we can search out by one way or the other a soft positive integer n_0 which behaves as $x_n \in (\mathcal{G}, \mathcal{A})$ for all $n \geqslant n_0$ that is the soft $\alpha-$open set $(\mathcal{G}, \mathcal{A})$ absorbing l contains all except finite number of terms of the soft sequence $\{x_n\}$ but on the other hand $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi$. So $(\mathcal{H}, \mathcal{A})$ contains all those terms of the given soft sequence which are left from $(\mathcal{G}, \mathcal{A})$ So the set $(\mathcal{H}, \mathcal{A})$ containing m contains finite number of terms of the soft sequence $\{x_n\}$. This leads us to the result that $\{x_n\}$ cannot converge to (m). But this is purely a contradiction to the fact that $\{x_n\}$ converges to m. This contradiction is taking birth due to our wrong supposition that either $l > m$ or $l < m$. Hence we are constrained to accept that l coincides m. This qualifies us to say that a soft convergent sequence in soft $\alpha-$Hausdorff space has unique soft limit. □

Theorem 4.29. *In a soft $\beta-$Hausdorff space every soft sequence converges to at most one point.*

Proof. Suppose $(\mathcal{X}, \tau, \mathcal{A})$ be a soft $\beta-$Hausdorff space and $\{x_n\}$ be a soft sequence approaches the soft limit l. We prove this result by contradiction. Suppose the limit of the soft sequence is not unique and converges to another soft real number say m which is entirely different of l. Since $(\mathcal{X}, \tau, \mathcal{A})$ be a soft $\beta-$Hausdorff space. So by definition we can find two soft mutually exclusive $\beta-$open sets $(\mathcal{G}, \mathcal{A})$ and $(\mathcal{H}, \mathcal{A})$ such that $l \in (\mathcal{G}, \mathcal{A})$ and $m \in (\mathcal{H}, \mathcal{A})$ such that $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi$. Since x_n converges to l so we can search out by one way or the other a soft positive integer n_0 which behaves as $x_n \in (\mathcal{G}, \mathcal{A})$ for all $n \geqslant n_0$ that is the soft $\beta-$open set $(\mathcal{G}, \mathcal{A})$ absorbing l contains all except finite number of terms of the soft sequence $\{x_n\}$ but on the other hand $(\mathcal{G}, \mathcal{A}) \cap (\mathcal{H}, \mathcal{A}) = \phi$. So $(\mathcal{H}, \mathcal{A})$ contains all those terms of the given soft sequence which are left from $(\mathcal{G}, \mathcal{A})$ So the set $(\mathcal{H}, \mathcal{A})$ containing m contains finite number of terms of the soft sequence $\{x_n\}$. This leads us to the result that $\{x_n\}$ cannot converge to (m). But this is purely a contradiction to the fact that $\{x_n\}$ converges to m. This contradiction is taking birth due to our wrong supposition that either $l > m$ or $l < m$. Hence we are constrained to accept that l coincides m. This qualifies us to say that a soft convergent sequence in soft $\beta-$Hausdorff space has unique soft limit. □

5. conclusion

Topology is the most important branch of mathematics which deals with mathematical buildings. Recently, many scholars have studied the soft set theory and their applications in mathematics. M. Shabir and M. Naz [6] introduced and deeply studied the conception of soft topological spaces. They also studied soft topological structures and exhibited their several properties with respect to ordinary points In this present work, we have continued to study the concept of soft limit point in soft bi topological space is introduced and related results are also discussed with respect to ordinary and soft points. Soft interior point in soft bi topological space and related results with respect to ordinary and soft points are also studied. The direct bridge between soft weak T_0 space and Soft weak closure is touched in soft topological space with respect to soft weak open sets. Soft neighborhood in soft bi topological space is defined and related results are studied. Soft sequences uniqueness of limit in soft weak Hausdorff space is studied. The product of soft Hausdorff spaces with respect to ordinary and soft points in different soft weak open sets are also discussed. The linkage between Soft Hausdorff space and the diagonal is also planted here These results would be useful for the development of the theory of soft topology to solve knotty problems, hugging doubts in economics, engineering, medical etc. We also beautifully discussed some soft transmissible properties with respect to ordinary as well as soft points. This research will build strongly the foundation of soft bi topological spaces. We hope that these results will help the researchers for strengthening the toolbox of soft topology. In the future, we extend the notion of soft $\alpha-$open, soft $\beta-$open and soft P open sets to soft $b^{(**)}$ open sets in soft bi topological spaces with respect to ordinary as well as soft points.

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